RATIONAL & IRRATIONAL NUMBERS

Ex.3. Rationalising factor of $3\sqrt{2}$ is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$

(*c*) $3\sqrt{3}$ (*d*) None of these

Sol. (*a*) $\sqrt{2}$

Ex.4. Rationalising factor of $4\sqrt{2} - 1$ is

(*a*) $4\sqrt{2} - 1$ (*b*) $4\sqrt{2} + 1$

(*c*) $4\sqrt{5} - 1$ (*d*) None of these

Sol. (*b*) $4\sqrt{2} + 1$

Ex.5. Rationalising factor of $4\sqrt{5} - 3\sqrt{2}$ is

(*a*) $4\sqrt{5} + 3\sqrt{2}$ (*b*) $4\sqrt{5} - 3\sqrt{2}$

(*c*) $2\sqrt{5} + \sqrt{2}$ (*d*) None of these

Sol. (*a*) $4\sqrt{5} + 3\sqrt{2}$

Ex.6. Rationalise the denominator of the following:

(*a*) $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$ (*b*) $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}}$

Sol. (*a*) $\dfrac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \dfrac{(\sqrt{7}-\sqrt{5})(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}$

$= \dfrac{7+5-2\sqrt{7}\cdot\sqrt{5}}{7-5} = \dfrac{12-2\sqrt{35}}{2}$

$= \dfrac{2(6-\sqrt{35})}{2} = 6 - \sqrt{35}$

(*b*) $\dfrac{1}{\sqrt{6}+\sqrt{5}-\sqrt{11}} =$

$\dfrac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{[(\sqrt{6}+\sqrt{5})-(\sqrt{11})][(\sqrt{6}+\sqrt{5})+(\sqrt{11})]}$

$= \dfrac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{(\sqrt{6}+\sqrt{5})^2-11} = \dfrac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{6+5+2\sqrt{30}-11}$

$= \dfrac{\sqrt{6}+\sqrt{5}+\sqrt{11}}{2\sqrt{30}}$

$= \dfrac{(\sqrt{6}+\sqrt{5}+\sqrt{11})\sqrt{30}}{2\times\sqrt{30}\times\sqrt{30}}$

$= \dfrac{\sqrt{180}+\sqrt{150}+\sqrt{330}}{60}$

$= \dfrac{6\sqrt{5}+5\sqrt{6}+\sqrt{330}}{60}$

Ex.7. Simplify:

(*a*) $\dfrac{4+\sqrt{5}}{4-\sqrt{5}} + \dfrac{4-\sqrt{5}}{4+\sqrt{5}}$

(*b*) $\left(\dfrac{1}{\sqrt{5}-2} - \dfrac{1}{\sqrt{5}+2}\right)\left(\dfrac{1}{2+\sqrt{3}} + \dfrac{1}{2-\sqrt{3}}\right)$

Sol. (*a*) $\dfrac{4+\sqrt{5}}{4-\sqrt{5}} + \dfrac{4-\sqrt{5}}{4+\sqrt{5}}$

$= \dfrac{(4+\sqrt{5})^2+(4-\sqrt{5})^2}{16-5}$

$= \dfrac{16+5+8\sqrt{5}+16+5-8\sqrt{5}}{11}$

$= \dfrac{42}{11} = 3\dfrac{9}{11}$

(*c*) $\left(\dfrac{1}{\sqrt{5}-2} - \dfrac{1}{\sqrt{5}+2}\right)\left(\dfrac{1}{2+\sqrt{3}} + \dfrac{1}{2-\sqrt{3}}\right)$

$= \left\{\dfrac{\sqrt{5}+2-(\sqrt{5}-2)}{5-4}\right\}\left(\dfrac{2-\sqrt{3}+2+\sqrt{3}}{4-3}\right)$

$= \left(\dfrac{\sqrt{5}+2-\sqrt{5}+2}{1}\right)\left(\dfrac{4}{1}\right) = \dfrac{4}{1}\times\dfrac{4}{1}$

$= 16$

Ex.8. If $x = 3 - 2\sqrt{2}$, find $x^2 + \dfrac{1}{x^2}$.

Sol. Given $x = 3 - 2\sqrt{2}$

then, $x^2 = (3 - 2\sqrt{2})^2 = 9 + 8 - 12\sqrt{2}$

$= 17 - 12\sqrt{2}$

Now $x^2 + \dfrac{1}{x^2} = \dfrac{17-12\sqrt{2}}{1} + \dfrac{1}{17-12\sqrt{2}}$

$= \dfrac{(17-12\sqrt{2})^2+1}{17-12\sqrt{2}}$

$= \dfrac{289+288-408\sqrt{2}+1}{17-12\sqrt{2}}$

$= \dfrac{578-408\sqrt{2}}{17-12\sqrt{2}}$

$= \dfrac{(578-408\sqrt{2})(17+12\sqrt{2})}{(17-12\sqrt{2})(17+12\sqrt{2})}$

$= \dfrac{(578-408\sqrt{2})(17+12\sqrt{2})}{289-288}$

$= \dfrac{(578-408\sqrt{2})(17+12\sqrt{2})}{1}$

$= 9826 + 6936\sqrt{2} - 6936\sqrt{2} - 9792$

$= 34$

Ex.9. If $a = 1 - \sqrt{3}$, find the value of $\left(a - \dfrac{1}{a}\right)^3$.

Sol. $a = 1 - \sqrt{3} \Rightarrow \dfrac{1}{a} = \dfrac{1}{1-\sqrt{3}}$

$\therefore\ a - \dfrac{1}{a} = (1-\sqrt{3}) - \dfrac{1}{1-\sqrt{3}}$

$$= \frac{1+3-2\sqrt{3}-1}{1-\sqrt{3}} = \frac{3-2\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{(3-2\sqrt{3})(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$$

$$= \frac{3+3\sqrt{3}-2\sqrt{3}-6}{1-3} = \frac{\sqrt{3}-3}{-2}$$

Now, $\left(a-\frac{1}{a}\right)^3 = \left(\frac{\sqrt{3}-3}{-2}\right)^3 = \left(\frac{3-\sqrt{3}}{2}\right)^3$

$$= \frac{3^3-(\sqrt{3})^3-3\times 3\times\sqrt{3}(3-\sqrt{3})}{2^3}$$

$$= \frac{27-3\sqrt{3}-9\sqrt{3}\,(3-\sqrt{3})}{8}$$

$$= \frac{27-3\sqrt{3}-27\sqrt{3}+27}{8}$$

$$= \frac{54-30\sqrt{3}}{8} = \frac{27-15\sqrt{3}}{4}$$

Ex.10. Determine rational numbers x and y. if

(*a*) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = x - \sqrt{y}$

(*b*) $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = x + y\sqrt{7}$

Sol. (*a*) $\frac{4+\sqrt{2}}{2+\sqrt{2}} = x - \sqrt{y}$

$$\Rightarrow \frac{(4+\sqrt{2})(2-\sqrt{2})}{(2+\sqrt{2})(2-\sqrt{2})} = x - \sqrt{y}$$

$$\Rightarrow \frac{8+2\sqrt{2}-4\sqrt{2}-2}{4-2} = x - \sqrt{y}$$

$$\Rightarrow \frac{6-2\sqrt{2}}{2} = x - \sqrt{y}$$

$$\Rightarrow 3 - \sqrt{2} = x - \sqrt{y}$$

$$\Rightarrow x = 3 \text{ and } y = 2$$

(*b*) $\frac{\sqrt{7}-1}{\sqrt{7}+1} - \frac{\sqrt{7}+1}{\sqrt{7}-1} = x + y\sqrt{7}$

$$\Rightarrow \frac{(\sqrt{7}-1)^2-(\sqrt{7}+1)^2}{(\sqrt{7}-1)(\sqrt{7}+1)} = x + y\sqrt{7}$$

$$\Rightarrow \frac{7+1-2\sqrt{7}-(7+1+2\sqrt{7})}{(7-1)} = x + y\sqrt{7}$$

$$\Rightarrow \frac{-4\sqrt{7}}{6} = x + y\sqrt{7}$$

$$\Rightarrow x = 0 \text{ and } y = \frac{-2}{3}$$

PRACTICE QUESTIONS

1. When we rationalise the denominator of $\frac{2+\sqrt{3}}{2-\sqrt{3}}$, we get

   (*a*) $7 + 4\sqrt{3}$ (*b*) $4\sqrt{3} - 7$

   (*c*) $7 - 4\sqrt{3}$ (*d*) None of these

2. If $x = \frac{3+\sqrt{2}}{3-\sqrt{2}}$ then $x + \frac{1}{x} =$

   (*a*) $3\frac{3}{7}$ (*b*) $3\frac{1}{7}$

   (*c*) $3\frac{2}{7}$ (*d*) None of these

3. If $x = \frac{\sqrt{3}+2\sqrt{2}}{\sqrt{3}-2\sqrt{2}}$ then $x - \frac{1}{x}$ is

   (*a*) $4\frac{1}{5}$ (*b*) $-4\frac{2}{5}$

   (*c*) $4\frac{2}{5}$ (*d*) None of these

4. If $p + q = \sqrt{6}$ and $p - q = \sqrt{2}$, then pq is

   (*a*) 1 (*b*) 0

   (*c*) –1 (*d*) None of these

5. If $a + b = \sqrt{7}$ and $a - b = \sqrt{3}$, then $a^2 + b^2$ is

   (*a*) 4 (*b*) 5

   (*c*) –4 (*d*) None of these

6. Rationalise the denominator of the following:

   (*a*) $\frac{1}{5\sqrt{3}-3\sqrt{5}}$ (*b*) $\frac{2}{\sqrt{5}+\sqrt{3}-2\sqrt{2}}$

7. Simplify:

   (*a*) $\frac{3+\sqrt{2}}{3-\sqrt{2}} + \frac{3-\sqrt{2}}{3+\sqrt{2}}$

   (*b*) $\frac{1}{\sqrt{5}+\sqrt{3}} - \frac{2}{\sqrt{5}+\sqrt{2}} + \frac{2}{\sqrt{3}+\sqrt{2}}$

8. If x, y and z are rational numbers and $\frac{3}{1+\sqrt{3}-\sqrt{7}} = x + y\sqrt{3} + z\sqrt{7} + w\sqrt{21}$.

   Find the value of x, y, z and w.

9. If $x = 5 + 3\sqrt{2}$, find $x^2 + \frac{1}{x^2}$.

10. If $a = 2 + \sqrt{3}$, find $a^3 + \frac{1}{a^3}$.

Expansion of $(a \pm b)^2$

Ex.1. $(a + 3b)^2$ **in expanded form is**

(*a*) $a^2 + 9b^2 + 6ab$ (*b*) $a^2 - 9b^2 - 6ab$

(*c*) $a^2 + 9b^2 - 6ab$ (*d*) None of these

Sol. (*a*) $a^2 + 9b^2 + 6ab$

Ex.2. $(2a - 5b)^2$ **in expanded form is**

(*a*) $a^2 + 25b^2 - 20ab$

(*b*) $4a^2 - 25b^2 + 20ab$

(*c*) $4a^2 + 25b^2 - 20ab$

(*d*) None of these

Sol. (*c*) $(2a - 5b)^2 = (2a)^2 + (5b)^2 - 2 \times 2a \times 5b$

$= 4a^2 + 25b^2 - 20ab$

Ex.3. **The value of** $(x + 2y)^2 + (x - 2y)^2$ **is**

(*a*) $2x^2 + 8y^2$ (*b*) $2x^2 - 8y^2$

(*c*) $x^2 - 4y^2$ (*d*) None of these

Sol. (*a*) $(x + 2y)^2 + (x - 2y)^2 = 2x^2 + 8y^2$

Ex.4. **If** $x + y = 16$ **and** $x - y = 2$ **then value of** xy **is**

(*a*) 60 (*b*) 163

(*c*) –63 (*d*) None of these

Sol. (*d*) On solving, we get $xy = 9 \times 7 = 63$

Ex.5. **Expand the following:**

(*a*) $(2x + y)^2$ (*b*) $(3x - 2y)^2$

Sol. (*a*) $(2x + y)^2 = (2x)^2 + (y)^2 + 2 \times 2x \times y$

$= 4x^2 + y^2 + 4xy$

(*b*) $(3x - 2y)^2 = (3x)^2 + (2y)^2 - 2 \times 3x \times 2y$

$= 9x^2 + 4y^2 - 12xy$

Ex.6. **Expand:** $\left(\frac{1}{2}x + \frac{2}{3}y\right)^2$.

Sol. $\left(\frac{1}{2}x + \frac{2}{3}y\right)^2 = \left(\frac{1}{2}x\right)^2 + \left(\frac{2}{3}y\right)^2 + 2 \times \frac{1}{2}x \times \frac{2}{3}y$

$= \frac{1}{4}x^2 + \frac{4}{9}y^2 + \frac{2}{3}xy$

Ex.7. **Expand:** $(3x^2y + 5z)^2$

Sol. $(3x^2y + 5z)^2 = (3x^2y)^2 + (5z)^2 + 2 \times 3x^2y \times 5z$

$= 9x^4y^2 + 25z^2 + 30x^2yz$

Ex.8. **If** $a - b = 8$ **and** $ab = 5$**, find** $a^2 + b^2$**.**

Sol. $a - b = 8$ and $ab = 5$

We know that $a^2 + b^2 = (a - b)^2 + 2ab$

$\Rightarrow a^2 + b^2 = 8^2 + 2 \times 5$

$= 64 + 10$

$\therefore a^2 + b^2 = 74$

Ex.9. **If the sum and the product of two numbers are 7 and** $\frac{45}{4}$ **respectively, find the sum of their cubes.**

Sol. Let a and b be the numbers.

$\therefore a + b = 7$ and $ab = \frac{45}{4}$

$\therefore a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$= 7^3 - 3 \times \frac{45}{4}(7)$

$= 343 - \frac{945}{4} = \frac{1372 - 945}{4}$

$= \frac{427}{4} = 106\frac{3}{4}$

Ex.10. Use $(a + b)^2 = a^2 + 2ab + b^2$ to evaluate the following:

(*i*) $(102)^2$ (*ii*) $(1002)^2$

(*iii*) $(10.2)^2$

Sol. (*i*) $(102)^2 = (100 + 2)^2$
$= (100)^2 + (2)^2 + 2 \times 100 \times 2$
$= 10000 + 4 + 400$
$= 10404$

(*ii*) $(1002)^2 = (1000 + 2)^2$
$= (1000)^2 + (2)^2 + 2 \times 1000 \times 2$
$= 1000000 + 4 + 4000$
$= 1004004$

(*iii*) $(10.2)^2 = (10 + 0.2)^2$
$= (10)^2 + (0.2)^2 + 2 \times 10 \times 0.2$
$= 100 + 0.04 + 4 = 104.04$

Ex.11. Use $(a - b)^2 = a^2 - 2ab + b^2$ to evaluate the following:

(*i*) 99^2 (*ii*) $(999)^2$

(*iii*) $(9.9)^2$

Sol. (*i*) $99^2 = (100 - 1)^2$
$= (100)^2 + (1)^2 - 2 \times 100 \times 1$
$= 10000 + 1 - 200 = 9801$

(*ii*) $(999)^2 = (1000 - 1)^2$
$= (1000)^2 + (1)^2 - 2 \times 1000 \times 1$
$= 1000000 + 1 - 2000$
$= 998001$

(*iii*) $(9.9)^2 = (10 - 0.1)^2$
$= (10)^2 + (0.1)^2 - 2 \times 10 \times 0.1$
$= 100 + 0.01 - 2 = 98.01$

Ex.12. If $x + y = 1$ and $x - y = 7$, find the values of (*a*) $5(x^2 + y^2)$ (*b*) xy.

Sol. $x + y = 1$ and $x - y = 7$

$\Rightarrow (x + y)^2 - (x - y)^2 = 4xy$

$\Rightarrow 1^2 - 7^2 = 4xy$

$\Rightarrow 1 - 49 = 4xy$

$\Rightarrow 4xy = -48$

$\Rightarrow xy = -12$ ...(*i*)

Now, we know that

$x^2 + y^2 = (x + y)^2 - 2xy$
$= 1^2 - 2 \times (-12)$
$= 1 + 24 = 25$

(*a*) $5(x^2 + y^2) = 25 \times 5 = 125$

(*b*) $xy = -12$ [Using eq. (*i*)]

PRACTICE QUESTIONS

1. If $a + b = 12$ and $ab = 35$ then value of $(a - b)^2$ is
(*a*) 1 (*b*) 2
(*c*) 4 (*d*) None of these

2. If $p^2 + \frac{1}{p^2} = \frac{17}{4}$, then the value of $p + \frac{1}{p}$ is
(*a*) $\pm \frac{5}{2}$ (*b*) $\pm \frac{3}{2}$
(*c*) $\pm \frac{7}{2}$ (*d*) None of these

3. If $a^2 - \frac{1}{a^2} = 7$, then value of $a^4 + \frac{1}{a^4}$ is
(*a*) 49 (*b*) 51
(*c*) 52 (*d*) None of these

4. If $x + \frac{1}{x} = 4$, then $\left(x - \frac{1}{x}\right)^2$ is equal to
(*a*) 6 (*b*) 10
(*c*) 12 (*d*) None of these

5. If $p^2 + \frac{1}{p^2} = 23$, then the value of $p + \frac{1}{p}$ is
(*a*) 2 (*b*) ± 5
(*c*) ± 4 (*d*) None of these

6. By using standard formula, expand the following:
(*a*) $\left(\frac{x}{2} + \frac{3y}{2}\right)^2$ (*b*) $(3x - 5y)^2$
(*c*) $(3x^2 + 5y^2)^2$ (*d*) $(4a^3 - 7b^3)^2$

7. Use $(a + b)^2 = a^2 + 2ab + b^2$ to evaluate the following:
(*a*) $(104)^2$ (*b*) $(1005)^2$
(*c*) $(10.7)^2$

8. Simplify $(2a - b)^2 + (2a + b)^2$

9. Simplify $\left(3x + \frac{1}{3x}\right)^2 + \left(3x - \frac{1}{3x}\right)^2$

Ex.3. The coefficients of x^2 and the constant term in the product of $(x + 1)(x - 4)(x + 5)$ are

(*a*) 2, 20 (*b*) –19, 20
(*c*) 19, – 20 (*d*) 2, –20

Sol. (*d*) $(x + 1)(x - 4)(x + 5)$
$= (x^3 - 3x - 4)(x + 5)$
$= x^3 + 2x^2 - 19x - 20$
$\therefore$ coefficient of x^2 and the constant term are 2 and –20 respectively.

Ex.4. Find the coefficient of x^2 and x in the product $(x + 2)(x + 4)(x + 7)$

Sol. $(x + 2)(x + 4)(x + 7) = x^3 + (2 + 4 + 7)x^2 + (2 \times 4 + 4 \times 7 + 2 \times 7)x + 2 \times 4 \times 7$
$= x^3 + 13x^2 + (8 + 28 + 14)x + 56$
$= x^3 + 13x^2 + 50x + 56$
Coefficient of $x^2 = 13$; Coefficient of $x = 50$

Ex.5. Find the coefficient of x^2 and x in the product $(x - 1)(x + 3)(x - 4)$

Sol. $(x - 1)(x + 3)(x - 4)$
$a = -1;\ b = 3;\ c = -4$
Coefficient of $x^2 = a + b + c = -1 + 3 - 4 = -2$
Coefficient of $x = ab + bc + ca = (-1) \times 3 + 3 \times (-4) + (-1) \times (-4) = -3 - 12 + 4 = -11$

PRACTICE QUESTIONS

1. The coefficient of x^2 in the product $(x + 1)(x + 2)(x + 3)$ is
(*a*) 5 (*b*) 6
(*c*) 4 (*d*) None of these
2. The coefficient of x in the product $(x - 2)(x + 3)(x - 5)$ is
(*a*) 10 (*b*) 11
(*c*) –11 (*d*) None of these
3. The coefficient of x^2 and x in the product of $(x - 6)(x + 7)(x - 8)$ are respectively
(*a*) 7, 50 (*b*) –7, –50
(*c*) 8, 50 (*d*) None of these
4. Find the coefficient of x^2 and x in the product $(x - 3)(x - 4)(x - 8)$
5. Find the coefficient of x^2 and x in the product $(x - 4)(x + 7)(x - 9)$

Expansion of $(a \pm b)^3$

Ex.1. $\left(2x + \frac{3}{2}y\right)^3$ in expanded form is

(*a*) $8x^3 + \frac{27}{8}y^3 + 18x^2y + \frac{27xy^2}{2}$

(*b*) $8x^3 - \frac{27}{8}y^3 + 18x^2y + \frac{27xy^2}{2}$

(*c*) $8x^3 + \frac{27}{8}y^3 - 18x^2y + \frac{27xy^2}{2}$

(*d*) None of these

Sol. (*a*) $\left(2x + \frac{3}{2}y\right)^3 = (2x)^3 + \left(\frac{3}{2}y\right)^3 + 3 \times 2x \times \frac{3}{2}y \times \left(2x + \frac{3}{2}y\right)$

$= 8x^3 + \frac{27}{8}y^3 + 18x^2y + \frac{27xy^2}{2}.$

Ex.2. $\left(x - \frac{1}{2x}\right)^3$ in expanded form is

(*a*) $x^3 - \frac{1}{8x^3} + \frac{3x}{2} + \frac{3}{4x}$

(*b*) $x^3 - \frac{1}{8x^3} - \frac{3x}{2} + \frac{3}{4x}$

(*c*) $x^3 - \frac{1}{8x^3} - \frac{3x}{2} - \frac{3}{4x}$

(*d*) None of these

Sol. (*b*) $x^3 - \frac{1}{8x^3} - \frac{3x}{2} + \frac{3}{4x}$

Ex.3. If $x - \frac{1}{x} = 2$, then $x^3 - \frac{1}{x^3}$ is equal to

(*a*) 12 (*b*) 14
(*c*) 10 (*d*) None of these

Sol. (*b*) 14

Ex.4. If $\frac{x}{y} + \frac{y}{x} = 1$, then $x^3 + y^3$ is equal to

(*a*) 0 (*b*) 1
(*c*) –1 (*d*) None of these

Sol. (*a*) 0

Ex.5. If $x - y = 2$ and $x^3 - y^3 = 68$ then xy is equal to

(*a*) 10 (*b*) 0
(*c*) 5 (*d*) None of these

Sol. (*a*) 10

Ex.6. If $a + b = 4$ and $a^3 + b^3 = 52$, then value of ab is

(*a*) –1 (*b*) 1
(*c*) 4 (*d*) None of these

Sol. (*b*) 1

Ex.7. **If $x^2 + 1 = 4x$ then to value of $x^3 + \frac{1}{x^3}$ is**

(a) 42 (b) 52

(c) –42 (d) None of these

Sol. (b) 52

Ex.8. **If $x + 3y = 5$, prove that $x^3 + 27y^3 + 45xy = 125$**

Sol. Given; $x + 3y = 5$

On cubing both sides, we have

$$(x + 3y)^3 = 5^3$$

$$\Rightarrow x^3 + (3y)^3 + 3 \times x \times 3y\ (x + 3y) = 125$$

$$\Rightarrow x^3 + 27y^3 + 9xy\ (x + 3y) = 125$$

$$\Rightarrow x^3 + 27y^3 + 9xy \times 5 = 125 \quad [\because x + 3y = 5]$$

$$\Rightarrow x^3 + 27y^3 + 45xy = 125$$ **Hence proved.**

Ex.9. **If $p + q = 1 + pq$, prove that $p^3 + q^3 = 1 + p^3q^3$**

Sol. We know, $p^3 + q^3 = (p + q)^3 - 3pq\ (p + q)$

$= (1 + pq)^3 - 3pq\ (1 + pq)$

$= 1 + p^3q^3 + 3pq\ (1 + pq) - 3pq\ (1 + pq)$

$= 1 + p^3q^3$

$\therefore p^3 + q^3 = 1 + p^3q^3$ **Hence proved.**

Ex.10. **Solve: $(2x + 3y)^3$**

Sol. $(2x + 3y)^3 = (2x)^3 + (3y)^3 + 3 \times 2x \times 3y\ (2x + 3y)$

$= 8x^3 + 27y^3 + 18xy\ (2x + 3y)$

$= 8x^3 + 27y^3 + 36x^2y + 54xy^2$

Ex.11. **If $a + \frac{1}{a} = p$, prove that $a^3 + \frac{1}{a^3} = p(p^2 - 3)$.**

Sol. Given; $a + \frac{1}{a} = p$

On cubing both sides, we have

$$\left(a + \frac{1}{a}\right)^3 = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3 \times a \times \frac{1}{a}\left(a + \frac{1}{a}\right) = p^3$$

$$\Rightarrow a^3 + \frac{1}{a^3} + 3\,(p) = p^3 \Rightarrow a^3 + \frac{1}{a^3} = p^3 - 3p$$

$$\therefore a^3 + \frac{1}{a^3} = p\,(p^2 - 3).$$ **Hence proved.**

Ex.12. **If $\frac{x^2 + 1}{x} = 4$, find the value of $2x^3 + \frac{2}{x^3}$**

Sol. $\frac{x^2 + 1}{x} = 4 \Rightarrow x^2 + 1 = 4x$

$$\Rightarrow x^2 - 4x + 1 = 0 \quad ...(i)$$

On dividing equation (i) by x, we have

$$x - 4 + \frac{1}{x} = 0$$

$$\Rightarrow x + \frac{1}{x} = 4 \quad ...(ii)$$

On cubing equation (ii) we have

$$\left(x + \frac{1}{x}\right)^3 = (4)^3$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right) = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 4 = 64$$

$$\Rightarrow x^3 + \frac{1}{x^3} = 64 - 12 = 52$$

$$\therefore 2\left(x^3 + \frac{1}{x^3}\right) = 2 \times 52 = 104$$

$$\Rightarrow 2x^3 + \frac{2}{x^3} = 104$$

PRACTICE QUESTIONS

1. If $x + \frac{1}{x} = 3$, then $x^6 + \frac{1}{x^6}$ is equal to

(a) 302 (b) 312

(c) 322 (d) None of these

2. If $2x + \frac{2}{x} = 3$, then value of $x^3 + \frac{1}{x^3} + 2$

(a) $\frac{4}{7}$ (b) $\frac{5}{8}$

(c) $\frac{7}{8}$ (d) None of these

3. If $x + y = 5$, then $x^3 + y^3 + 15xy$ is equal to

(a) 225 (b) 125

(c) 25 (d) None of these

4. If $a - b = 7$ and $a^2 + b^2 = 85$ then $a^3 - b^3$ is equal to

(a) 722 (b) 712

(c) 721 (d) None of these

5. If $x + y = 7$ and $x^3 + y^3 = 133$ then $x^2 + y^2$ is equal to

(a) 30 (b) 28

(c) 29 (d) None of these

6. $\dfrac{0.9\times0.9\times0.9-0.6\times0.6\times0.6}{0.9\times0.9+0.9\times0.6+0.6\times0.6}$ is equal to

(a) 0.4 (b) 0.3

(c) 0.6 (d) None of these

7. Expand the following by using standard formula.

(a) $(2a + b)^3$ (b) $(2x - 1)^3$

(c) $(4p - 7q)^3$ (d) $\left(4p-\dfrac{1}{3q}\right)^3$

8. If $2x - \dfrac{1}{2x} = y$, find $8x^3 - \dfrac{1}{8x^3}$

9. If $r + \dfrac{1}{r} = \sqrt{3}$, prove that $r^3+\dfrac{1}{r^3} = 0$

10. If $x^2 + \dfrac{1}{x^2} = 5$, find $x^3 + \dfrac{1}{x^3}$

11. If $x^2 - 5x + 1 = 0$, find $x^3 + \dfrac{1}{x^3}$

12. If $x + 2y = 7$, prove that $x^3 + 8y^3 + 42xy - 343 = 0$

13. The sum of the numbers a and b is 9, whereas their product is 20. Find

(i) $a^3 + b^3$ (ii) $a^2 - ab + b^2$

14. Solve: $\left(3x+\dfrac{1}{x}\right)^3$.

Expansion of $(a \pm b \pm c)^2$

Ex.1. $(x + 2y - 3z)^2$ in expanded form is

(a) $x^2 + 4y^2 - 9z^2 + 4xy + 12yz + 6xz$

(b) $x^2 + 4y^2 + 9z^2 + 4xy - 12yz - 6xz$

(c) $x^2 + 4y^2 + 9z^2 + 4xy + 12yz - 6xz$

(d) None of these

Sol. (b) $x^2 + 4y^2 + 9z^2 + 4xy - 12yz - 6xz$

Ex.2. $(2a - 3b - 4c)^2$ in expanded form is

(a) $4a^2 + 9b^2 + 16c^2 - 12ab + 24bc - 16ac$

(b) $4a^2 + 9b^2 - 16c^2 - 12ab - 24bc + 16ac$

(c) $4a^2 - 9b^2 - 16c^2 - 12ab + 24bc + 16ac$

(d) None of these

Sol. (a) $4a^2 + 9b^2 + 16c^2 - 12ab + 24bc - 16ac$

Ex.3. If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 29$, then the value of $ab + bc + ca$ is

(a) 22 (b) 24

(c) 26 (d) None of these

Sol. (c) 26

Ex.4. Expand: $(a + 2b + c)^2$

Sol. $(a + 2b + c)^2 = a^2 + (2b)^2 + c^2 + 2 \times a \times 2b + 2 \times 2b \times c + 2 \times a \times c$

$= a^2 + 4b^2 + c^2 + 4ab + 4bc + 2ac$

Ex.5. Expand: $\left(x+\dfrac{1}{x}-1\right)^2$

Sol. $\left(x+\dfrac{1}{x}-1\right)^2 = \left[x+\dfrac{1}{x}+(-1)\right]^2$

$= (x)^2 + \left(\dfrac{1}{x}\right)^2 + (-1)^2 + 2 \times x \times \dfrac{1}{x} + 2 \times \dfrac{1}{x} \times (-1) + 2 \times x \times (-1)$

$= x^2 + \dfrac{1}{x^2} + 1 + 2 - \dfrac{2}{x} - 2x$

$= x^2 + \dfrac{1}{x^2} - \dfrac{2}{x} - 2x + 3$

Ex.6. Expand: $\left(\dfrac{2}{3}x-\dfrac{3}{2x}-1\right)^2$

Sol. $\left(\dfrac{2}{3}x-\dfrac{3}{2x}-1\right)^2 = \left[\dfrac{2x}{3}+\left(\dfrac{-3}{2x}\right)+(-1)\right]^2$

$= \left(\dfrac{2x}{3}\right)^2 + \left(\dfrac{-3}{2x}\right)^2 + (-1)^2 + 2 \times \dfrac{2x}{3} \times \left(\dfrac{-3}{2x}\right) + 2 \times \left(\dfrac{-3}{2x}\right) \times (-1) + 2 \times \dfrac{2x}{3} \times (-1)$

$= \dfrac{4x^2}{9} + \dfrac{9}{4x^2} + 1 - 2 + \dfrac{3}{x} - \dfrac{4x}{3}$

$= \dfrac{4x^2}{9} + \dfrac{9}{4x^2} + \dfrac{3}{x} - \dfrac{4x}{3} - 1$

Ex.7. If $a + b + c = 12$ and $a^2 + b^2 + c^2 = 100$, find $ab + bc + ca$.

Sol. $a + b + c = 12$ and $a^2 + b^2 + c^2 = 100$

We know that,

$$(a+b+c)^2 = a^2+b^2+c^2 + 2(ab+bc+ca)$$

$$\Rightarrow \quad (12)^2 = 100 + 2(ab+bc+ca)$$

$$\Rightarrow \quad 144 - 100 = 2(ab+bc+ca)$$

$$\Rightarrow \quad \frac{44}{2} = ab+bc+ca$$

$$\Rightarrow \quad ab+bc+ca = 22$$

PRACTICE QUESTIONS

1. If $a + 2b - c = 5$ and $a^2 + 4b^2 + c^2 = 41$, then $2ab - 2bc - ac$ is

   (*a*) –8 (*b*) –6
   (*c*) 8 (*d*) None of these

2. If $a + b + c = 15$ and $a^2 + b^2 + c^2 = 77$, then $ab + bc + ca$ is

   (*a*) 76 (*b*) 64
   (*c*) 74 (*d*) None of these

3. If $a + 2b - 3c = 4$ and $2ab - 6bc - 3ac = -9$, then value of $a^2 + 4b^2 + 9c^2$ is

   (*a*) 43 (*b*) 34
   (*c*) 36 (*d*) None of these

4. Expand the following by using standard formulae.

   (*a*) $\left(2x - \frac{1}{2x} + 1\right)^2$ (*b*) $\left(\frac{3x}{4} - \frac{3}{5x} - 2\right)^2$

5. If $a + b + c = 14$ and $a^2 + b^2 + c^2 = 50$, find $ab + bc + ca$

6. If $a^2 + b^2 + c^2 = 14$ and $ab + bc + ca = 11$, find $a + b + c$

7. If $x + y - z = 5$ and $x^2 + y^2 + z^2 = 29$, find the value of $xy - yz - zx$

8. If $3x - y + z = 0$, prove that $9x^2 + z^2 - y^2 + 6xz = 0$

INTEGRATED (MIXED) QUESTIONS

1. The expression $(x + 4y)(2x - 1)$ in expanded form is

   (*a*) $3x^2 - x + 8xy - 4y$ (*b*) $2x^2 + 8xy - 4y$
   (*c*) $3x^2 + 4xy$ (*d*) $2x^2 - x + 8xy - 4y$

2. $(a^2 + 3b)^2$ is in expanded form is

   (*a*) $a^4 + 9b^2 + 6a^2b$ (*b*) $a^4 + b^2 + 6a^2b$
   (*c*) $a^2 + 9b^2 + 6a^2b$ (*d*) $a^4 + 9b^2 - 6a^2b$

3. The simplified value of $\left(2x - \frac{1}{2x}\right)^2 - \left(2x + \frac{1}{2x}\right)\left(2x - \frac{1}{2x}\right)$ is

   (*a*) $\frac{1}{2x^2} + 2$ (*b*) $\frac{1}{2x^2} - 2$
   (*c*) $\frac{1}{x}\left(2x - \frac{1}{x}\right)$ (*d*) $2x - \frac{1}{2x}$

4. The simplified value of $(5x + 3y)^2 - (3x - 5y)^2$ is

   (*a*) $60xy$ (*b*) $25x^2 - 9y^2 + 60xy$
   (*c*) $16x^2 - 16y^2 + 60xy$ (*d*) $16x^2 + 60xy$

5. If $p + q = 10$ and $pq = 16$, then the value of $3(p^2 + q^2)$ is

   (*a*) 68 (*b*) 204
   (*c*) 96 (*d*) None of these

6. Simplify: $(1 - x)(1 + x)(1 + x^2)(1 + x^4)$

7. Simplify: $(a - b)^2 + (a + b)^2$

8. Simplify: $(a + b)^2 - (a - b)^2$

9. Simplify: $\left(x + \frac{1}{x}\right)^2 + \left(x - \frac{1}{x}\right)^2$

10. Simplify: $(3p - 1)^2 - (3p - 2)(3p + 1)$

11. If the sum of two numbers a and b is 7 and their product is 12, find $a^2 - ab + b^2$.

12. If $2a + 3b = 7$ and $ab = 2$, find $4a^2 + 9b^2$.

13. If $x + y = 6$ and $x - y = 4$, find
    (*a*) $x^2 + y^2$, (*b*) xy.

14. If $x^2 + y^2 = 34$ and $xy = \frac{21}{2}$, find the value of $2(x + y)^2 + (x - y)^2$.

15. If $a = \frac{1}{a-5}$, find

    (*i*) $a - \frac{1}{a}$ (*ii*) $a + \frac{1}{a}$ (*iii*) $a^2 - \frac{1}{a^2}$

16. Solve: $(x - 2)(x - 3)(x + 4)$

17. If $a^2 + 4a + x = (a + 2)^2$, find the value of x

18. If $2x - y + z = 0$, prove that $4x^2 - y^2 + z^2 + 4xz = 0$

Use of Laws of Exponents

Ex.1. If $9 \times 81^x = \dfrac{1}{27^{x-3}}$ then x is equal to

(*a*) 0 (*b*) 1
(*c*) –1 (*d*) None of these

Sol. (*b*)
$$9 \times 81^x = \frac{1}{27^{x-3}}$$
$$\Rightarrow \quad 3^2 \times 3^{4x} = \frac{1}{3^{3x-9}}$$
$$\Rightarrow \quad 3^{4x+2} = 3^{-3x+9}$$
On comparing powers,
$$\Rightarrow \quad 4x + 2 = -3x + 9$$
$$\Rightarrow \quad x = 1$$

Ex.2. If $2^{x+1} + 2^x = 3$ then $3^x + 3^{-x}$ is equal to

(*a*) 0 (*b*) 1
(*c*) 2 (*d*) None of these

Sol. (*c*)
$$2^{x+1} + 2^x = 3$$
$$\Rightarrow \quad 2^x(2 + 1) = 3$$
$$\Rightarrow \quad 2^x = 1$$
$$\Rightarrow \quad 2^x = 2^0 \Rightarrow x = 0$$
$$\therefore \quad 3^x + 3^{-x} = 3^0 + 3^{-0}$$
$$= 1 + \frac{1}{1} = 2$$

Ex.3. If $4^y = 8^x$ then $y : x$ is

(*a*) 2 : 3 (*b*) 3 : 2
(*c*) 3 : 1 (*d*) None of these

Sol. (*b*)
$$4^y = 8^x$$
$$\Rightarrow \quad 2^{2y} = 2^{3x}$$
$$\therefore \quad 3x = 2y$$
$$\Rightarrow \quad \frac{3}{2} = \frac{y}{x}$$
$$y : x = 3 : 2$$

Ex.4. If $(2^5 + 0.125)^2 - (2^5 - 0.125)^2 = 2^x$ then value of x is

(*a*) –4 (*b*) 4
(*c*) 2 (*d*) None of these

Sol. (*b*) $(2^5 + 0.125)^2 - (2^5 - 0.125)^2 = 2^x$
$$\Rightarrow \quad \left(2^5+\frac{1}{8}\right)^2-\left(2^5-\frac{1}{8}\right)^2 = 2^x$$
$$\Rightarrow \quad \left(2^5+\frac{1}{2^3}\right)^2-\left(2^5-\frac{1}{2^3}\right)^2 = 2^x$$
$$\Rightarrow \quad 2^{10}+\frac{1}{2^6}+2\times 2^5\times\frac{1}{2^3} - \left[2^{10}+\frac{1}{2^6} - 2\times 2^5\times\frac{1}{2^3}\right] = 2^x$$
$$\Rightarrow \quad 2^{10}+\frac{1}{2^6}+2^3-2^{10}-\frac{1}{2^6}+2^3 = 2^x$$
$$\Rightarrow \quad 2.2^3 = 2^x$$
$$\Rightarrow \quad 2^4 = 2^x$$
On comparing powers,
$$x = 4$$

Ex.5. If $8^{2x+1} = 16^{x+1}$ then x is equal to

(*a*) $\frac{1}{2}$ (*b*) 2
(*c*) 1 (*d*) None of these

Sol. (*a*)
$$8^{2x+1} = 16^{x+1}$$
$$\Rightarrow \quad (2^3)^{2x+1} = (2^4)^{x+1}$$
$$\Rightarrow \quad 2^{6x+3} = 2^{4x+4}$$
$$\Rightarrow \quad 6x + 3 = 4x + 4$$
$$\Rightarrow \quad x = \frac{1}{2}$$

Ex.6. Prove that $\sqrt{x^{-1}.y}\,.\sqrt{y^{-1}.z}\,.\sqrt{z^{-1}.x} = 1$

Sol. LHS $= \sqrt{x^{-1}.y}\,.\sqrt{y^{-1}.z}\,.\sqrt{z^{-1}.x}$
$$= \sqrt{x^{-1}.y.y^{-1}.z.z^{-1}.x}$$
$$= \sqrt{x^0.y^0.z^0} = \sqrt{1} = 1 = \text{RHS}$$

Ex.7. Prove that $\left(\dfrac{x^m}{x^n}\right)^l.\left(\dfrac{x^n}{x^l}\right)^m.\left(\dfrac{x^l}{x^m}\right)^n = 1$

Sol. LHS $= \left(\dfrac{x^m}{x^n}\right)^l.\left(\dfrac{x^n}{x^l}\right)^m.\left(\dfrac{x^l}{x^m}\right)^n$
$$= (x^{m-n})^l.(x^{n-l})^m.(x^{l-m})^n$$
$$= x^{lm-ln}.x^{nm-ml}.x^{ln-mn}$$
$$= x^{lm-ln+mn-ml+ln-mn}$$
$$= x^0 = 1 = \text{RHS}$$

Ex.8. Prove that

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=1$$

Sol. LHS $= \dfrac{1}{1+x^{b-a}+x^{c-a}}+\dfrac{1}{1+x^{a-b}+x^{c-b}}+\dfrac{1}{1+x^{b-c}+x^{a-c}}$

$$= \frac{1}{1+\frac{x^b}{x^a}+\frac{x^c}{x^a}}+\frac{1}{1+\frac{x^a}{x^b}+\frac{x^c}{x^b}}+\frac{1}{1+\frac{x^b}{x^c}+\frac{x^a}{x^c}}$$

$$= \frac{1}{\frac{x^a+x^b+x^c}{x^a}}+\frac{1}{\frac{x^b+x^a+x^c}{x^b}}+\frac{1}{\frac{x^c+x^b+x^a}{x^c}}$$

$$= \frac{x^a}{x^a+x^b+x^c}+\frac{x^b}{x^a+x^b+x^c}+\frac{x^c}{x^a+x^b+x^c}$$

$$= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1 = \text{RHS}$$

Ex.9. Show that $\dfrac{\left(p+\frac{1}{q}\right)^m \times \left(p-\frac{1}{q}\right)^n}{\left(q+\frac{1}{p}\right)^m \times \left(q-\frac{1}{p}\right)^n} = \left(\dfrac{p}{q}\right)^{m+n}$.

Sol. LHS $= \dfrac{\left(p+\frac{1}{q}\right)^m \times \left(p-\frac{1}{q}\right)^n}{\left(q+\frac{1}{p}\right)^m \times \left(q-\frac{1}{p}\right)^n}$

$$= \frac{\left(\frac{pq+1}{q}\right)^m \times \left(\frac{pq-1}{q}\right)^n}{\left(\frac{pq+1}{p}\right)^m \times \left(\frac{pq-1}{p}\right)^n}$$

$$= \frac{\frac{(pq+1)^m . (pq-1)^n}{q^m . q^n}}{\frac{(pq+1)^m}{p^m} \times \frac{(pq-1)^n}{p^n}} = \frac{p^m . p^n}{q^m . q^n}$$

$$= \left(\frac{p}{q}\right)^{m+n} = \text{RHS}$$

Ex.10. Show that $\sqrt[bc]{\dfrac{x^b}{x^c}} . \sqrt[ca]{\dfrac{x^c}{x^a}} . \sqrt[ab]{\dfrac{x^a}{x^b}} = 1$

Sol. LHS $= \sqrt[bc]{\dfrac{x^b}{x^c}} . \sqrt[ca]{\dfrac{x^c}{x^a}} . \sqrt[ab]{\dfrac{x^a}{x^b}}$

$$= (x^{b-c})^{\frac{1}{bc}} . (x^{c-a})^{\frac{1}{ca}} . (x^{a-b})^{\frac{1}{ab}}$$

$$= x^{\frac{1}{c}-\frac{1}{b}} . x^{\frac{1}{a}-\frac{1}{c}} . x^{\frac{1}{b}-\frac{1}{a}}$$

$$= x^{\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}+\frac{1}{b}-\frac{1}{a}} = x^0 = 1$$

$= \text{RHS}$

Ex.11. Solve for x and y:

(*a*) $3^x \times 3^{-y} = 9$ and $2^y \times 4^{-x} = \dfrac{1}{8}$

(*b*) $3^x = y^2$ and $y^x = 9$

Sol. (*a*) $3^x \times 3^{-y} = 9$

$\Rightarrow \quad 3^{x-y} = 3^2$

$\Rightarrow \quad x - y = 2$ ...(*i*)

and $\quad 2^y \times 4^{-x} = \dfrac{1}{8}$

$\Rightarrow \quad 2^y \times 2^{-2x} = 2^{-3}$

$\Rightarrow \quad 2^{-2x+y} = 2^{-3}$

$\Rightarrow \quad -2x + y = -3$ ...(*ii*)

Adding (*i*) and (*ii*), we get

$$\begin{array}{r} x - y = 2 \\ -2x + y = -3 \\ \hline -x = -1 \end{array} \quad \Rightarrow x = 1$$

Substituting the value of x in (*i*), we get

$x - y = 2$

$\Rightarrow \quad 1 - y = 2$

$\Rightarrow \quad -y = 1$

$\Rightarrow \quad y = -1$

(*b*) $\quad 3^x = y^2$

$\Rightarrow \quad y = 3^{\frac{x}{2}}$ ...(*i*)

Now $\quad y^x = 9$

$\Rightarrow \quad \left(3^{\frac{x}{2}}\right)^x = 9$ [from (*i*)]

$\Rightarrow \quad 3^{\frac{x^2}{2}} = 3^2$

$\Rightarrow \quad \dfrac{x^2}{2} = 2$

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$

$$y = 3^{\frac{x}{2}} = 3^{\frac{2}{2}}$$

$\Rightarrow$ $y = 3$

or $y = 3^{\frac{-2}{2}}$

$$y = \frac{1}{3}$$

Ex.12. Solve the following equations and find the value of x and value of y (wherever possible)

(*a*) $3^x + 3^{-x} = 2$

(*b*) $4^{2x} = (\sqrt[3]{16})^{-6/y} = (\sqrt{8})^2$

(*c*) $8^{x+1} = 16^{y+2}, \left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$

Sol. (*a*) $3^x + 3^{-x} = 2 \Rightarrow 3^x + \frac{1}{3^x} = 2$

On putting $3^x = t$, we get

$$t + \frac{1}{t} = 2$$

$$\Rightarrow t^2 + 1 = 2t \Rightarrow t^2 - 2t + 1 = 0$$

$$\Rightarrow (t-1)^2 = 0 \Rightarrow t = 1$$

$$\Rightarrow 3^x = 1 = 3^0 \Rightarrow x = 0$$

(*b*) $4^{2x} = (\sqrt[3]{16})^{-6/y} = (\sqrt{8})^2$

$$\Rightarrow 4^{2x} = 8 \Rightarrow 2^{4x} = 2^3$$

$$\Rightarrow 4x = 3 \Rightarrow x = \frac{3}{4}$$

and $(\sqrt[3]{16})^{-6/y} = 8 \Rightarrow 16^{\frac{-2}{y}} = 2^3$

$$\Rightarrow 2^{\frac{-8}{y}} = 2^3 \Rightarrow -\frac{8}{y} = 3$$

$$\Rightarrow y = \frac{-8}{3}$$

(*c*) $8^{x+1} = 16^{y+2}$

$$\Rightarrow 2^{3x+3} = 2^{4y+8}$$

$$\Rightarrow 3x + 3 = 4y + 8$$

$$\Rightarrow 3x - 4y = 5 \quad ...(i)$$

and $\left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{4}\right)^{3y}$

$$\Rightarrow \left(\frac{1}{2}\right)^{3+x} = \left(\frac{1}{2}\right)^{6y}$$

$$\Rightarrow 3 + x = 6y$$

$$\Rightarrow x - 6y = -3 \quad ...(ii)$$

on multiplying eq (*ii*) by 3 and subtracting from (*i*), we get

$$3x - 4y = 5$$
$$3x - 18y = -9$$
$$- \quad + \qquad +$$
$$14y = 14 \Rightarrow y = 1$$

By putting the value of y in (*ii*), we get

$$x - 6 = -3 \Rightarrow x = 3$$

Ex.13. Solve for x and y: $3^{x-y} = 27; 3^{x+y} = 243$

Sol. $3^{x-y} = 27 = 3^3$

$$\Rightarrow x - y = 3 \quad ...(i)$$

and $3^{x+y} = 243 = 3^5$

$$\Rightarrow x + y = 5 \quad ...(ii)$$

By adding (*i*) and (*ii*), we get

$$x - y = 3$$
$$x + y = 5$$
$$2x = 8 \Rightarrow x = 4$$

By putting the value of x in (*i*), we get

$$4 - y = 3 \Rightarrow y = 1$$

Ex.14. If $p^{1/x} = q^{1/y} = r^{1/z}$ and $pqr = 1$. Prove that $x + y + z = 0$.

Sol. $p^{1/x} = q^{1/y} = r^{1/z} = k$ (say)

$$\Rightarrow p = k^x, q = k^y, r = k^z$$

Given, $pqr = 1$

$$\Rightarrow k^x . k^y . k^z = 1 \Rightarrow k^{(x+y+z)} = k^0$$

$\Rightarrow x + y + z = 0$ Hence proved

Ex.15. If $a = 2 + 2^{2/3} + 2^{1/3}$. Prove that $a^3 - 6a^2 + 6a - 2 = 0$.

Sol. Given $a = 2 + 2^{2/3} + 2^{1/3}$

$$a - 2 = 2^{2/3} + 2^{1/3}$$

$$\Rightarrow (a-2)^3 = (2^{2/3} + 2^{1/3})^3$$

$$\Rightarrow a^3 - 8 - 3 \times a \times 2(a-2)$$
$$= (2^{2/3})^3 + (2^{1/3})^3 + 3 \times 2^{2/3} \times 2^{1/3}(2^{2/3} + 2^{1/3})$$

$$\Rightarrow a^3 - 8 - 6a(a-2) = 2^2 + 2^1 + 3 \times 2^1(a-2)$$

$$\Rightarrow a^3 - 8 - 6a^2 + 12a = 4 + 2 + 6a - 12$$

$\Rightarrow a^3 - 6a^2 + 6a - 2 = 0$ Hence proved

Triangles and properties of triangles

Ex.1. In ΔABC, $AB \neq BC \neq AC$. ΔABC is a/an

(a) isosceles triangle
(b) scalene triangle
(c) equilateral triangle
(d) None of these

Sol. (b) scalene triangle

Ex.2. In ΔABC, $AB = BC \neq AC$. ΔABC is a/an

(a) isosceles triangle
(b) scalene triangle
(c) equilateral triangle
(d) None of these

Sol. (a) isosceles triangle

Ex.3. In ΔABC, $\angle C = 90°$, ΔABC is a/an

(a) otuse angled triangle
(b) right-angled triangle
(c) acute angled triangle
(d) None of these

Sol. (b) right-angled triangle

Ex.4. In ΔABC, $\angle A : \angle B : \angle C = 1 : 2 : 3$. Find the angles and identify the type of the triangle.

Sol. $\angle A : \angle B : \angle C = 1 : 2 : 3$

Let $\angle A$, $\angle B$ and $\angle C$ be x, $2x$ and $3x$ respectively.

We have,

$\angle A + \angle B + \angle C = 180°$ [Sum of angles of a Δ is 180°]

$x + 2x + 3x = 180°$

$6x = 180° \Rightarrow x = 30°$

$\angle A = x = 30°$;

$\angle B = 2x = 2 \times 30° = 60°$;

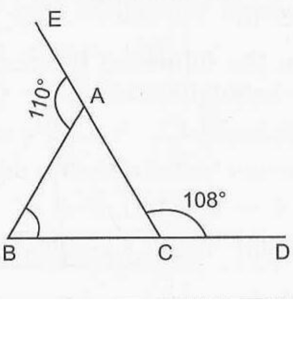
$\angle C = 3x = 3 \times 30° = 90°$

So, $\angle A = 30°, \angle B = 60°$ and $\angle C = 90°$

Hence, ΔABC is a right-angled triangle at point C.

Ex.5. ABC is a triangle, in which BC is produced to D, CA is produced to E, $\angle DCA = 108°$ and $\angle BAE = 110°$. Calculate $\angle ABC$.

Sol.



$\angle ACB = 180° - 108°$ [Linear pair]

$\Rightarrow \angle ACB = 72°$

and $\angle BAC = 180° - 110° = 70°$ [Linear pair]

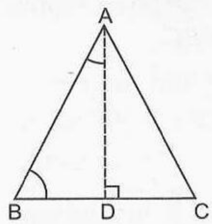
$\angle A + \angle B + \angle C = 180°$ [Sum of angles of a Δ is 180°]

$70° + \angle ABC + 72° = 180°$

$\angle ABC = 180° - 142° = 38°$

Ex.6. In an equilateral triangle ABC, the bisector of $\angle BAC$ meets BC at D. Find $\angle ADC$.

Sol.



$\angle ABC = 60°$ [An angle of an equilateral Δ]

$\angle BAD = 30°$ [AD is bisector of $\angle BAC$]

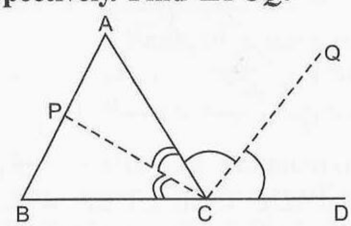
$\therefore \angle ADC = \angle ABD + \angle BAD$ [Exterior angle is equal to the sum of interior opposite angles]

$= 60° + 30° = 90°$

$\Rightarrow \angle ADC = 90°$

Ex.7. In a ΔABC, BC is produced to D. CP and CQ are bisectors of $\angle ACB$ and $\angle ACD$ respectively. Find $\angle PCQ$.

Sol.



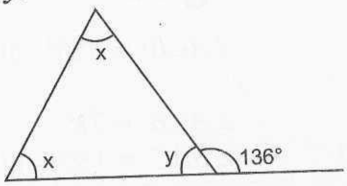
$\angle ACD + \angle ACB = 180°$ [Linear pair]

$\Rightarrow \frac{1}{2}\angle ACD + \frac{1}{2}\angle ACB = 90°$

$\Rightarrow \angle ACQ + \angle ACP = 90°$ [$\because$ CP and CQ are bisectors of $\angle ACB$ and $\angle ACD$]

$\Rightarrow \angle PCQ = 90°$ [$\because \angle ACQ + \angle ACP = \angle PCQ$]

Ex.8. From the given figure, find the values of x and y.



Sol. $y + 136° = 180°$ [Linear pair]

$\Rightarrow \quad y = 180° - 136° = 44°$

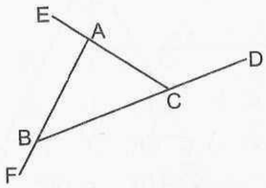
$136° = x + x$

[Exterior angle of a triangle is equal to sum of interior opposite angles]

$\Rightarrow \quad 2x = 136° \Rightarrow x = 68°$

$\therefore \quad x = 68°, y = 44°$

Ex.9. From the given figure, find $\angle ACD + \angle EAB + \angle FBC$.



Sol. $\angle BAC + \angle EAB + \angle ABC + \angle CBF + \angle ACB + \angle ACD = 540°$

[Three pairs of linear pair]

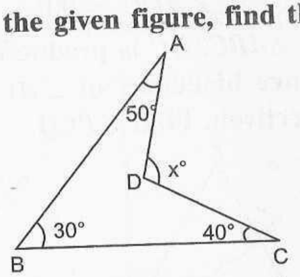
Now, $\angle EAB + \angle CBF + \angle ACD + (\angle ABC + \angle BAC + \angle ACB) = 540°$

$\angle EAB + \angle CBF + \angle ACD + 180° = 540°$

($\because$ sum of all interior angles of a Δ is 180°)

$\angle EAB + \angle CBF + \angle ACD = 360°$

Ex.10. From the given figure, find the value of x.



Sol. **Construction:** Join BD and produce to E.

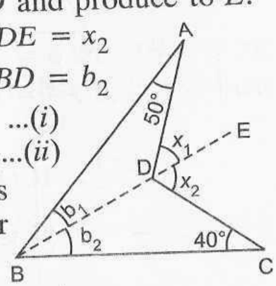
Let $\angle ADE = x_1$, $\angle CDE = x_2$

$\angle ABD = b_1$ and $\angle CBD = b_2$

$x_1 = b_1 + 50°$ ...(*i*)

$x_2 = b_2 + 40°$ ...(*ii*)

[Exterior angle of a Δ is equal to sum of interior opposite angles]

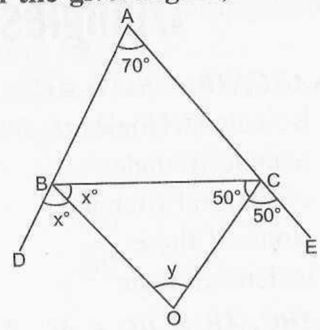


Adding (*i*) and (*ii*), we get

$x_1 + x_2 = b_1 + b_2 + 50° + 40°$

$\Rightarrow x = 30° + 90° \Rightarrow x = 120°$

Ex.11. From the given figure, find x and y.



Sol. $\angle ABC = 180° - 2x$ [Linear pair]

$\angle ACB = 180° - 100° = 80°$ [Linear pair]

In ΔABC,

$\angle ABC + \angle BAC + \angle ACB = 180°$

[Sum of angles of a Δ is 180°]

$\Rightarrow \quad 180° - 2x + 70° + 80° = 180°$

$\Rightarrow -2x = -150° \Rightarrow x = 75°$

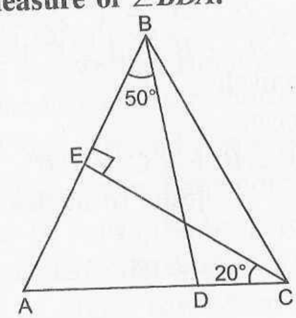
In ΔBOC, $x + y + 50° = 180°$

$\Rightarrow \quad 75° + y + 50° = 180°$

$y = 180° - 125° = 55$

$\Rightarrow \quad x = 75°, y = 55°$

Ex.12. In the given figure, CE is perpendicular to AB, $\angle ACE = 20°$ and $\angle ABD = 50°$. Find the measure of $\angle BDA$.



Sol. In ΔAEC,

$\angle CEA + \angle EAC + \angle ACE = 180°$

[Sum of angles of a Δ is 180°]

$90° + \angle EAC + 20° = 180°$

$\Rightarrow \quad \angle EAC = 70°$

In ΔABD,

$\angle ABD + \angle BDA + \angle BAD = 180°$

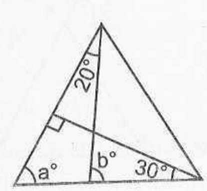
[Sum of angles of a Δ is 180°]

$50° + \angle BDA + 70° = 180°$

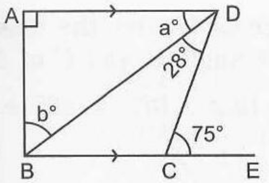
$\Rightarrow \quad \angle BDA = 60°$

Ex.13. In the following figures, find the values of a and b.

(*i*)



(*ii*)



Sol. (*i*) $a + 30° + 90° = 180°$

[Sum of angles of a Δ is 180°]

$a = 60°$

Also, $b = a + 20°$

[Exterior ∠*b* is equal to the sum of interior opposite angles]

$b = 60° + 20° = 80°$

Hence, $a = 60°, b = 80°$

(*ii*) $\angle DBC = \angle ADB = a°$

[Alternate angles]

$a + 28° = 75°$

[Exterior angle of a Δ is equal to the sum of interior opposite angles]

$a = 75° - 28° = 47°$

Also, $\angle B + \angle A = 180°$

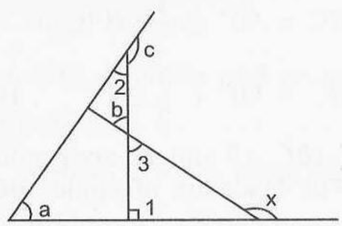
$\Rightarrow a + b + 90° = 180°$ [Sum of consecutive interior angles is 180°]

$47° + b + 90° = 180°$

$\Rightarrow b = 180° - 137° = 43°$

Hence, $a = 47°, b = 43°$

Ex.14. From the given figure, find the value of *x* in terms of *a*, *b* and *c*.



Sol. $\angle 2 + c = 180°$ [Linear pair]

$\angle 2 = 180° - c$

Also, $\angle 1 = a + \angle 2$

[Exterior angle is equal to the sum of interior opposite angles]

$\angle 1 = a + 180° - c$

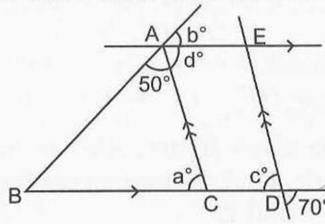
$\angle 3 = b$ [Vertically opposite angles]

Also, $x = \angle 1 + \angle 3$ [Exterior angle is equal to the sum of interior opposite angles]

$= a + 180° - c + b$

$x = a + b - c + 180°$

Ex.15. In the given figure, *AE* || *BC* and *CA* || *DE*. Find the values of *a* and *b*.



Sol. Let $\angle CDE = c°, \angle CAE = d°$

$c = 70°$

[Vertically opposite angles]

$a = c = 70°$

[Corresponding angles]

$a = 70°$

Also, $d = a = 70°$ [Alternate interior angles]

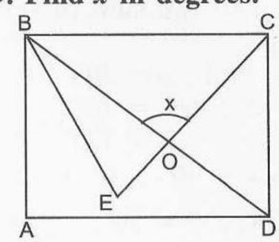
$d = 70°$

Now, $b + d + 50° = 180°$ [Straight angle]

$\Rightarrow b + 70° + 50° = 180° \Rightarrow b = 60°$

Hence, $a = 70°, b = 60°$

Ex.16. *BEC* is an equilateral triangle in square *ABCD*. Find *x* in degrees.



Sol. $\angle DBC = 45° = \angle OBC$

[*BD* is diagonal and it bisects ∠*ABC*]

$\angle BCE = 60° = \angle BCO$

[Angle of an equilateral triangle]

In $\Delta BOC, \angle CBO + \angle BCO + \angle BOC = 180°$

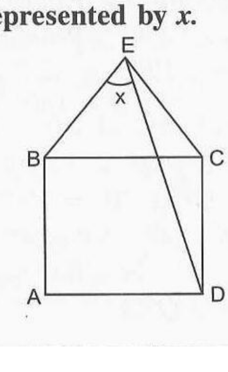
[Sum of angles of a Δ is 180°]

$\Rightarrow 45° + 60° + \angle BOC = 180°$

$\Rightarrow 105° + x = 180°$

$\Rightarrow x = 75°$

Ex.17. In the given figure, equilateral Δ*EBC* surmounts square *ABCD*. Find the angle *BED* represented by *x*.



Sol. In ΔECD, $EC = CD$ [Side of a square]

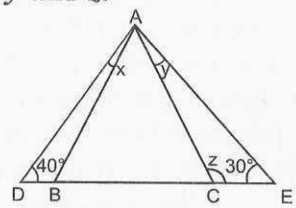
and $\angle ECD = 90° + 60° = 150°$

[$\angle BCE = 60°$, $\angle BCD = 90°$]

So, $\angle CED = \angle EDC = 15°$ [$CE = CD$]

$\therefore\ x = 60° - 15° = 45°$ [$\angle BEC = 60°$]

Ex.18. In the given figure, *ABC* is an equilateral triangle. Find the measures of angles marked by *x*, *y* and *z*.



Sol. ΔABC is an equilateral triangle [Given]

$\angle ABC = \angle ACB = \angle BAC = 60°$

In ΔABD, $\angle ABC = 40° + x$

[Exterior angle is equal to the sum of interior opposite angles]

$\Rightarrow\ 60° = 40° + x$

$\Rightarrow\ x = 20°$

In ΔACE, $60° = y + 30°$ [Exterior angle is equal to the sum of interior opposite angles]

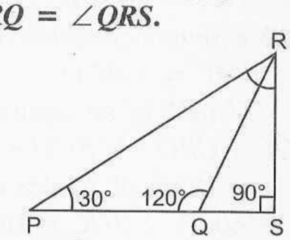
$\Rightarrow\ y = 30°$

Also, $z + 60° = 180°$ [Linear pair]

$\Rightarrow\ z = 120°$

Hence, $x = 20°, y = 30°, z = 120°$

Ex.19. In ΔPQR, $\angle P = 30°$, $\angle Q = 120°$ and *RS* is perpendicular to *PQ* produced. Show that $\angle PRQ = \angle QRS$.



Sol. Given: $RS \perp PQ$ produced, $\angle P = 30°$ and $\angle Q = 120°$

To prove: $\angle PRQ = \angle QRS$

Proof: In ΔPRQ,

$\angle RPQ + \angle PRQ + \angle PQR = 180°$

[Sum of angles of a Δ]

$\Rightarrow\ 30° + \angle PRQ + 120° = 180°$

$\Rightarrow\ \angle PRQ = 180° - 150° = 30°$

In ΔPRS,

$\angle PRS + \angle PSR + \angle RPS = 180°$

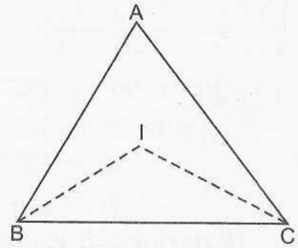
$\angle PRS + 90° + 30° = 180° \Rightarrow \angle PRS = 60°$

$\therefore\ \angle QRS = 60° - 30° = 30°$

[$\angle QRS = \angle PRS - \angle PRQ$]

$\angle PRQ = \angle QRS$ **Hence proved.**

Ex.20. In the given figure, the bisectors *BI* an[d] *CI* of the angle *B* and *C* of ΔABC meet i[n] *I*. Prove that $\angle BIC = 90° + \frac{\angle A}{2}$.



Sol. Given: *BI* and *CI* are bisectors of $\angle AB$[C] and $\angle ACB$.

To prove: $\angle BIC = 90° + \frac{\angle A}{2}$

Proof: In ΔABC,

$\angle ABC + \angle ACB + \angle BAC = 180°$

[Sum of $\angle$s of a Δ is 180°]

$\frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB + \frac{1}{2}\angle BAC = 90°$

$\frac{1}{2}\angle ABC + \frac{1}{2}\angle ACB = 90° - \frac{1}{2}\angle BAC$

$\angle IBC + \angle ICB = 90° - \frac{1}{2}\angle BAC$ ...(i)

[*BI* and *CI* are bisectors of $\angle B$ and $\angle C$]

In ΔBIC, $\angle BIC + \angle IBC + \angle ICB = 180$[°]

[Sum of $\angle$s of a Δ is 180°]

$\angle BIC + 90° - \frac{1}{2}\angle BAC = 180°$ [From (i)]

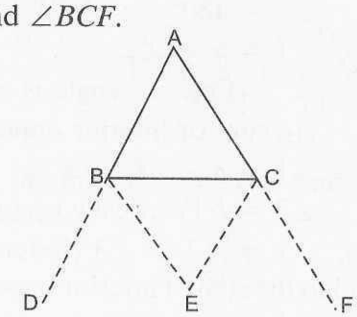
$\angle BIC = 180° - 90° + \frac{1}{2}\angle BAC$

$\angle BIC = 90° + \frac{1}{2}\angle BAC$

$\angle BIC = 90° + \frac{1}{2}\angle A$ **Hence prove[d]**

Ex.21. In ΔABC, *AB* and *AC* are produced to *D* an[d] *F*. The bisectors of angles *BCF* and *CB*[D] meet at *E*. Prove that $\angle BEC = 90° - \frac{\angle A}{2}$

Sol. Given: *BE* and *CE* are bisectors of $\angle DB$[C] and $\angle BCF$.



To prove: $\angle BEC = 90° - \frac{1}{2}\angle A$

Proof: $\angle DBC = 180° - \angle B$ [Linear pair]

$\angle BCF = 180° - \angle C$ [Linear pair]

$$\angle CBE = \frac{1}{2}\angle DBC$$

$$= \frac{1}{2}(180° - \angle B)$$

$$= 90° - \frac{1}{2}\angle B$$

$$\angle BCE = \frac{1}{2}\angle BCF$$

$$= \frac{1}{2}(180° - \angle C)$$

$$= 90° - \frac{1}{2}\angle C$$

Now, in ΔBEC

$\angle BEC + \angle ECB + \angle CBE = 180°$

[Sum of $\angle$s of a Δ]

$$\angle BEC + 90° - \frac{1}{2}\angle C + 90° - \frac{1}{2}\angle B = 180°$$

$$\angle BEC = \frac{1}{2}\angle B + \frac{1}{2}\angle C = \frac{1}{2}(\angle B + \angle C)$$

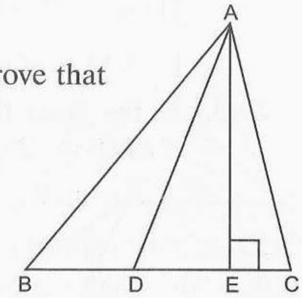
$$= \frac{1}{2}(180° - \angle A)$$

[$\angle B + \angle C = 180° - \angle A$]

$\angle BEC = 90° - \frac{1}{2}\angle A$ **Hence proved.**

23. The side BC of a ΔABC is produced to D; bisectors of the angles ABC and ACD meet at P. Prove that $\angle BPC = \frac{1}{2}\angle BAC$.

24. In the given fig., AD is bisector of $\angle A$ and AE is perpendicular to BC. Prove that $\angle DAE = \frac{1}{2}(\angle C - \angle B)$.



25. A transversal intersects two straight lines. If the bisectors of a pair of co-interior angles are perpendicular to each other, prove that the straight lines are parallel.

Congruency of triangles

Ex.1. If $\Delta ABC \cong \Delta PQR$, $AB = PQ$, $BC = QR$ and $\angle B = \angle Q$, then condition of congruency is

(*a*) *SAS* (*b*) *RHS*

(*c*) *AAS* (*d*) None of these

Sol. (*a*)

Ex.2. $\Delta ABC \cong \Delta PQR$, $\angle A = 40°$, $\angle B = 60°$, $AB = 5$ cm, $\angle P = 40°$, $\angle R = 80°$ and $PQ = 5$ cm. The condition of congruency is

(*a*) *RHS* (*b*) *AAS*

(*c*) *SAS* (*d*) None of these

Sol. (*b*)

Ex.3. Which of the following pairs of triangles are congruent? Give reason.

(*a*) $\Delta ABC : BC = 4$ cm, $CA = 5$ cm, $\angle C = 70°$
$\Delta PQR : PQ = 4$ cm, $QR = 5$ cm, $\angle Q = 70°$

(*b*) $\Delta ABC : AB = 4$ cm, $BC = 5$ cm, $\angle B = 70°$
$\Delta PQR : PQ = 4$ cm, $RP = 5$ cm, $\angle R = 70°$

(*c*) $\Delta ABC : AB = 5$ cm, $BC = 7$ cm, $CA = 9$ cm
$\Delta PQR : PQ = 7$ cm, $QR = 5$ cm, $RP = 9$ cm

Sol. (*a*) $\Delta ABC : BC = 4$ cm, $CA = 5$ cm, $\angle C = 70°$
$\Delta PQR : PQ = 4$ cm, $QR = 5$ cm, $\angle Q = 70°$

Yes, $\Delta BCA \cong \Delta PQR$

[SAS condition of congruency is satisfied]

(*b*) $\Delta ABC : AB = 4$ cm, $BC = 5$ cm, $\angle B = 70°$
$\Delta PQR : PQ = 4$ cm, $RP = 5$ cm, $\angle R = 70°$

ΔABC is not congruent to ΔPQR, because two sides are equal but included angle is not same.

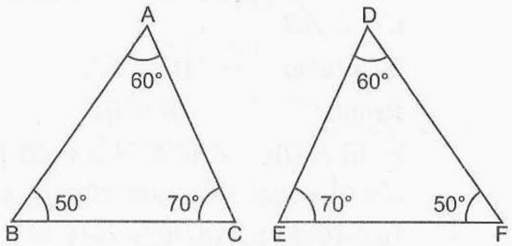
(*c*) $\Delta ABC : AB = 5$ cm, $BC = 7$ cm, $CA = 9$ cm
$\Delta PQR : PQ = 7$ cm, $QR = 5$ cm, $RP = 9$ cm

$\Delta ABC \cong \Delta RQP$

[SSS condition of congruency is satisfied]

Ex.4. ABC and DEF are two triangles in which $AB = DF$, $\angle ACB = 70°$, $\angle ABC = 50°$, $\angle DEF = 70°$ and $\angle EDF = 60°$. Prove that the two triangles are congruent.

Sol.



In ΔABC, $\angle BAC = 60°$ and in ΔDEF, $\angle DFE = 50°$

($\because$ Sum of angles of a triangle is 180°)

In ΔABC and ΔDEF,

$AB = DF$ [Given]

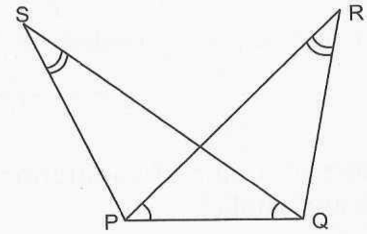
$\angle BAC = \angle EDF = 60°$

$\angle ACB = \angle DEF = 70°$ [Given]

Hence, $\Delta ABC \cong \Delta DFE$

[$\because$ *AAS*-condition of congruency is satisfied]

Ex.5. In the given figure, $\angle R = \angle S$ and $\angle RPQ = \angle PQS$. Prove that $PS = QR$.



Sol. Given: $\angle R = \angle S$ and $\angle RPQ = \angle PQS$

To prove: $PS = QR$

Proof: In ΔPQS and ΔPQR, we have

$PQ = PQ$ [Common]

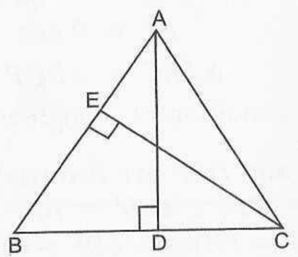
$\angle PSQ = \angle PRQ$ [Given]

$\angle RPQ = \angle PQS$ [Given]

Hence, $\Delta PQS \cong \Delta PQR$ [*AAS*]

$\therefore$ $PS = QR$ [*CPCT*]

Ex.6. In the given figure, $AB = BC$, $AD \perp BC$, $CE \perp AB$. Prove that $AD = CE$.



Sol. Given: $AB = BC$, $AD \perp BC$ and $CE \perp AB$ [Given]

To prove: $AD = CE$

Proof: $AB = BC$ [Given]

$\therefore$ In ΔABC, $\angle ACB = \angle CAB$ [$\because$ opposite $\angle$s of equal sides are equal]

In ΔACE and ΔADC, $\angle EAC = \angle ACD$ [$\angle ACB = \angle BAC$]

$\angle CEA = \angle ADC = 90°$

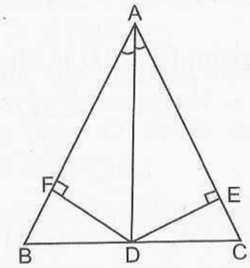
$AC = AC$ [AC is common]

Hence, $\Delta AEC \cong \Delta ADC$ [*AAS*]

$\therefore$ $AD = CE$ [*CPCT*]

Hence proved.

Ex.7. In the given figure, AD bisects $\angle A$, $DE \perp CA$ and $DF \perp AB$. Prove that $AF = AE$.



Sol. Given: AD is the bisector of $\angle BAC$. $DE \perp CA$ and $DF \perp AB$

To prove: $AF = AE$

Proof: In ΔAFD and ΔAED

$AD = AD$ [Common]

$\angle FAD = \angle DAE$

[$\because$ AD is bisector of $\angle FAE$]

$\angle AFD = \angle AED = 90°$

[$\because$ $DF \perp AB$ and $DE \perp AC$]

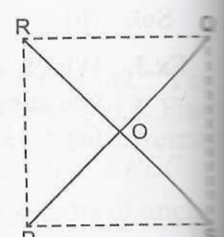
$\therefore$ $\Delta AFD \cong \Delta AED$ [AAS]

Hence, $AF = AE$ [*CPCT*]

Ex.8. Two line segments PQ and RS bisect each other at O. Prove that:

(a) $PR = QS$ (b) $\angle RPQ = \angle PQS$

(c) $PS \parallel RQ$ (d) $PS = RQ$



Sol. Given: PQ and RS bisect each other at O.

To prove: (*a*) $PR = QS$

(*b*) $\angle RPQ = \angle PQS$

(*c*) $PS \parallel RQ$ (*d*) $PS = RQ$

Proof: (*a*) In ΔPOR and ΔQOS, we have joined PR, PS, RQ and QS, $OR = OS$

$PO = OQ$ [Given]

$\angle POR = \angle QOS$ [Vertically opposite $\angle$s]

Hence, $\Delta POR \cong \Delta QOS$

[$\because$ *SAS*-condition of congruency is satisfied]

$\therefore$ $PR = QS$ [*CPCT*]

Hence proved.

(*b*) $\because$ $\Delta POR \cong \Delta QOS$

Hence, $\angle RPO = \angle OQS$ [*CPCT*]

$\Rightarrow$ $\angle RPQ = \angle PQS$

Hence proved.

(c) Similarly, $\Delta QOR \cong \Delta POS$

[$\because$ SAS-condition of congruency is satisfied]

$\therefore \quad \angle OQR = \angle OPS$ [CPCT]

Hence, $PS \parallel QR$. [Pair of alternate interior $\angle$s are equal]

(d) $\because \quad \Delta QOR \cong \Delta POS$

Hence, $PS = RQ$ [CPCT]

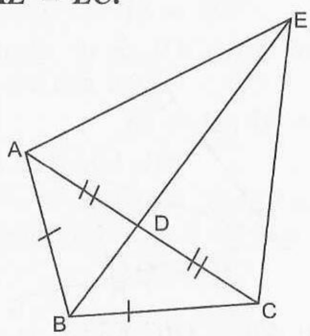
Hence proved.

Ex.9. In the given figure, $AB = BC$ and $AD = CD$.

Prove that:

(a) $\angle ADE$ = a right angle

(b) $AE = EC$.



Sol. Given: $AB = BC$ and $AD = DC$

To prove: (a) $\angle ADE = 90°$ (b) $AE = EC$

Proof: In ΔABD and ΔBCD, we have

$AB = BC$ [Given]

$AD = DC$ [Given]

$BD = BD$ [Common]

Hence, $\Delta ABD \cong \Delta BCD$

[$\because$ SSS-condition of congruency is satisfied]

(a) $\angle ADB = \angle CDB$ [CPCT]

But, $\angle ADB$ and $\angle CDB$ form a linear pair of angles.

$\therefore \quad \angle ADB = \angle CDB = 90°$

$\angle ADE = \angle CDB = 90°$

[Vertically opposite $\angle$s]

$\angle ADE = 90°$ **Hence proved.**

(b) In ΔADE and ΔCDE, we have DE is common

$AD = DC$ [Given]

$\angle ADE = \angle EDC = 90°$

So $\Delta ADE \cong \Delta CDE$

[$\because$ SAS-condition of congruency is satisfied]

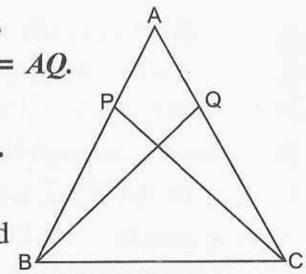
Hence, $AE = EC$ [CPCT] **Hence proved.**

Ex.10. In the given figure, $AB = AC$ and $AP = AQ$.

Prove that:

(a) $\Delta APC \cong \Delta AQB$.

(b) $\Delta BPC \cong \Delta CQB$.



Sol. Given: $AB = AC$ and $AP = AQ$

To prove: (a) $\Delta APC \cong \Delta AQB$ (b) $\Delta BPC \cong \Delta CQB$

Proof: $AB - AP = AC - AQ$ [$\because AP = AQ$]

i.e. $PB = QC$

(a) In ΔAPC and ΔAQB, we have

$AP = AQ$ [Given]

$\angle PAC = \angle BAQ$ [Common]

$AB = AC$ [Given]

Hence, $\Delta APC \cong \Delta AQB$

[$\because$ SAS-condition of congruency is satisfied]

$CP = BQ$ [CPCT]

Hence proved.

(b) In ΔBPC and ΔCQB, we have BC is common

$PB = CQ$ and $CP = BQ$ [Proved above]

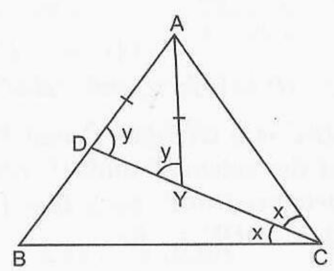
$\therefore \quad \Delta BPC \cong \Delta CQB$

[$\because$ SSS-condition of congruency is satisfied]

Hence proved.

Ex.11. ABC is a triangle. The bisector of the angle BCA meets AB in D. A point Y lies on CD such that $AD = AY$. Prove that $\angle CAY = \angle ABC$.

Sol.



Given: CD is the bisector of $\angle ACB$. $AD = AY$

To prove: $\angle CAY = \angle ABC$

Proof: Let $\angle ACD = \angle BCD = x$

and $\angle ADY = \angle AYD = y$

In ΔADC, $y + x + \angle DAC = 180°$

[Sum of all $\angle$s of a Δ is 180°]

$\angle DAC = 180° - (x + y)$

Also in ΔABC,

$\angle ABC + 2x + 180° - (x + y) = 180°$

$\angle ABC = y - x$ ...(*i*)

$\angle AYC = 180° - y$ [Linear pair]

In ΔAYC, $\angle YAC = 180° - [x + (180° - y)]$

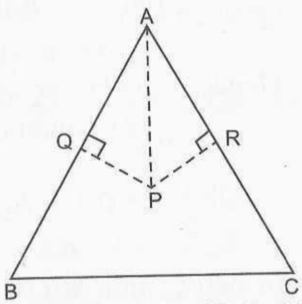
$\angle YAC = y - x$ ... (*ii*)

From equation (*i*) and (*ii*), $\angle CAY = \angle ABC$

Hence proved.

Ex.12. ***P* is any point in the ΔABC such that the perpendicular drawn from *P* on *AB* and *AC* are equal. Prove that *AP* is the bisector of $\angle BAC$.**

Sol.



Given: *P* is a point in ΔABC. $PQ \perp AB$ and $PR \perp AC$. Also $PQ = PR$

To prove: $\angle BAP = \angle CAP$

Proof: In ΔAPQ and ΔAPR, we have

AP is common.

$PQ = PR$

$\angle AQP = \angle ARP = 90°$

[Given]

Hence, $\Delta APQ \cong \Delta APR$

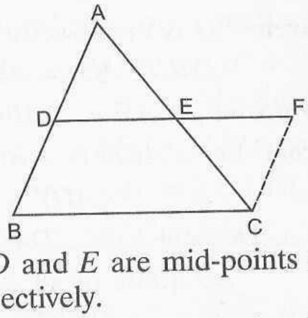
[$\because$ *RHS*-condition of congruency is satisfied]

$\angle QAP = \angle RAP$ [*CPCT*]

$\therefore$ *AP* is bisector of $\angle BAC$. **Hence proved.**

Ex.13. ***ABC* is a triangle. *D* and *E* are mid-points of the sides *AB* and *AC* respectively. *DE* is produced to *F*, such that *DE* = *EF*. Prove that $\Delta ADE \cong \Delta EFC$.**

Sol.



Given: *D* and *E* are mid-points of *AB* and *AC* respectively.

DE is produced to *F* such that *DE* = *EF*

To prove: $\Delta ADE \cong \Delta EFC$

Proof: In ΔADE and ΔEFC

$DE = EF$ [Given]

$\angle AED = \angle FEC$

[Vertically opposite $\angle$s]

$AE = EC$ [Given]

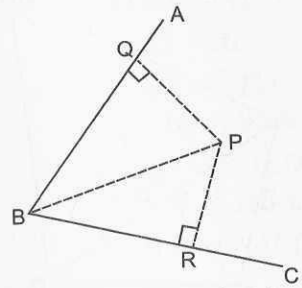
$\Delta ADE \cong \Delta EFC$

[$\because$ *SAS*-condition of congruency is satisfied]

Hence proved.

Ex.14. **Prove that any point on the bisector of an angle is equidistant from the arms of the angle.**

Sol.



Given: An $\angle ABC$ and *P* is any point on bisector of $\angle ABC$.

Construction: Draw $PQ \perp AB$ and $PR \perp BC$.

To prove: $PQ = PR$

Proof: In ΔBPQ and ΔBPR, we have

BP is common.

$\angle QBP = \angle PBR$

[*BP* is bisector of $\angle ABC$]

$\angle PQB = \angle PRB = 90°$

[Given]

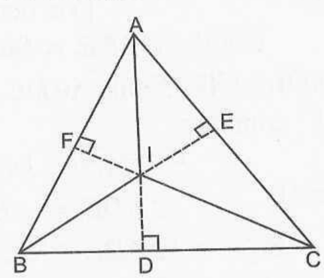
Hence, $\Delta BPQ \cong \Delta BPR$ [AAS]

$\therefore$ $PQ = PR$ [*CPCT*]

Hence proved.

Ex.15. **Prove that the internal bisectors of the angles of a triangle are concurrent.**

Sol.



Given: *ABC* is a triangle. *BI*, *CI* internal bisectors of $\angle B$ and $\angle C$ meet at *I*. *AI* bisects $\angle A$ internally.

To prove: Bisectors of $\angle A$, $\angle B$ and $\angle C$ meet at I, i.e., they are concurrent.

Construction: Draw ID, IE, IF perpendiculars on BC, CA and AB respectively.

Proof: In ΔIBF and ΔIBD.

$\angle IFB = \angle IDB = 90°$ [By construction]

$\angle IBD = \angle IBF$ [BI is bisector]

$BI = BI$ [Common]

$\therefore \quad \Delta IBF \cong \Delta IBD$

[$\because$ *AAS*-condition of congruency is satisfied]

$\therefore \quad IF = ID$ [*CPCT*]

Similarly, in Δs IDC, IEC and in Δs IAF, IAE, we can have,

$IE = ID, IF = IE$

In Δs AIF and AIE

$\angle AFI = \angle AEI = 90°$ [By construction]

$AI = AI$

$\therefore \quad \Delta AFI \cong \Delta AEI$

[$\because$ *RHS*-condition of congruency is satisfied]

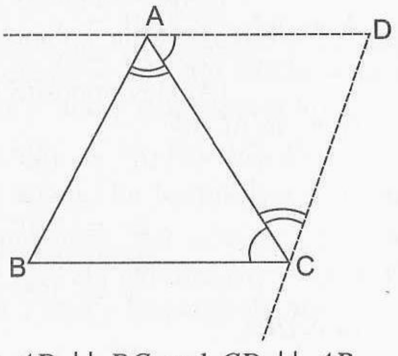
$\therefore \quad \angle FAI = \angle EAI$ [*CPCT*]

$\therefore$ AI bisects $\angle BAC$

Hence, internal bisectors of the angles of a Δ are concurrent. **Hence proved.**

Ex.16. The straight lines parallel to BC and BA drawn through the vertices A and C of ΔABC respectively, meet at point D. Prove that $\angle ABC = \angle CDA$.

Sol.



Given: $AD \parallel BC$ and $CD \parallel AB$

To prove: $\angle ABC = \angle CDA$

Proof: In ΔABC and ΔADC

$\angle DAC = \angle ACB$ [Alternate interior $\angle$s]

$\angle BAC = \angle ACD$ [Alternate interior $\angle$s]

AC is common.

Hence, $\quad \Delta ABC \cong \Delta ACD$

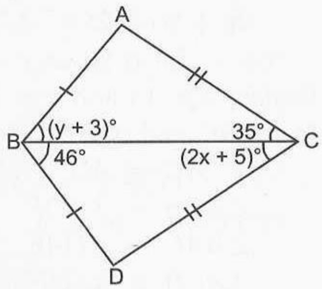
[$\because$ *ASA*-condition of congruency is satisfied]

So, $\quad \angle ABC = \angle ADC$ [*CPCT*]

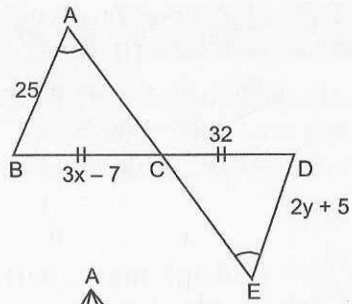
Hence proved.

Ex.17. In the following figures, find the values of x and y.

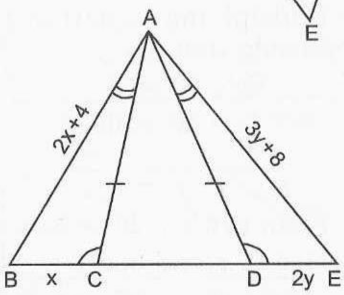
(*a*)



(*b*)



(*c*)



Sol. (*a*) In ΔABC and ΔDBC

$AB = BD$ [Given]

$AC = CD$ [Given]

BC is common

$\therefore \Delta ABC \cong \Delta DBC$

[$\because$ *SSS*-condition of congruency is satisfied]

Hence, $\angle ABC = \angle CBD$ [*CPCT*]

$y + 3 = 46° \Rightarrow y = 43°$

Also, $\angle ACB = \angle BCD$ [*CPCT*]

$35° = 2x + 5°$

$2x = 30° \Rightarrow x = 15°$

Hence, $x = 15°$ and $y = 43°$

(*b*) In ΔABC and ΔEDC

$\angle BAC = \angle CED$ [Given]

$BC = CD$

$\angle ACB = \angle DCE$ [Vertically opposite $\angle$s]

Hence, $\Delta ABC \cong \Delta CDE$

[$\because$ *AAS*-condition of congruency is satisfied]

Then, $3x - 7 = 32$ [*CPCT*]

$3x = 39 \Rightarrow x = 13$

$2y + 5 = 25$

$2y = 20 \Rightarrow y = 10$

Hence, $x = 13$ and $y = 10$

(*c*) In ΔABC and ΔADE,

$AC = AD$ [Given]

$\angle ACB = \angle ADE$ [Given]

$\angle BAC = \angle DAE$ [Given]

$\Delta ACB \cong \Delta ADE$

[$\because$ *ASA*-condition of congruency is satisfied]

$\therefore AB = AE$ [*CPCT*]

Then, $2x + 4 = 3y + 8$

$2x - 3y = 4$ ... (*i*)

Also, $BC = DE$

$x = 2y$

$x - 2y = 0$ ... (*ii*)

Multiplying equation (*ii*) by 2 and subtracting from (*i*),

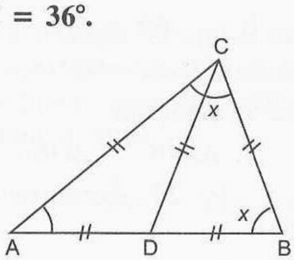
$$\begin{array}{r} 2x - 3y = 4 \\ 2x - 4y = 0 \\ - \quad + \qquad - \\ \hline y = 4 \end{array}$$

From (*ii*), $x - 2y = 0 \Rightarrow x = 8$

Hence, $x = 8$ and $y = 4$

Ex.18. In ΔABC, $AB = AC$ and D is a point on AB, such that $AD = DC = BC$. Show that $\angle BAC = 36°$.

Sol.



Given: $AB = AC$ and $AD = DC = BC$

To prove: $\angle BAC = 36°$

Proof: Let $\angle ABC = x$

So, $\angle ACB = x$ [$\because AB = AC$]

So, $\angle BAC = 180° - 2x$

[$\angle ABC + \angle BAC + \angle ACB = 180°$]

In ΔDBC, $BC = DC$ [Given]

So, $\angle BDC = \angle DBC = x$ [$BC = DC$]

Also, $AD = DC$

So, in ΔADC, $\angle DAC = \angle ACD = 180° - 2x$

$\angle ADC = 180° - x$

So, in ΔADC,

$180° - 2x + 180° - 2x + 180° - x = 180°$

[Sum of angles of a Δ is 180°]

$-5x = -360° \Rightarrow x = 72°$

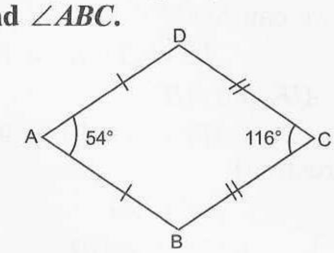
So, $\angle BAC = 180° - 2x$

$= 180° - 2 \times 72°$

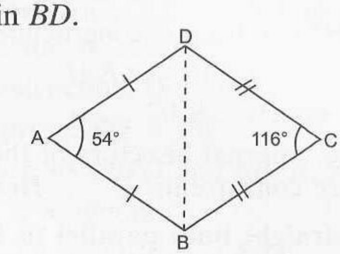
$= 180° - 144° = 36°$

Hence proved.

Ex.19. In the given figure, $AB = AD$, $BC = DC$. Find $\angle ABC$.



Sol. Join *BD*.



Let $\angle CBD = x$

$\therefore \angle CDB = x$

[Angles opposite to equal sides]

Also, let $\angle ABD = y$

$\angle ADB = y$

[Angles opposite to equal sides]

Now, in ΔCBD,

$x + x + 116° = 180°$

[Sum of all angles of a Δ is 180°]

$2x = 180° - 116°$

$2x = 64°$

$x = 32°$

In ΔABD,

$y + y + 54° = 180°$

[Sum of all angles of a Δ is 180°]

$2y = 180° - 54°$

$2y = 126°$

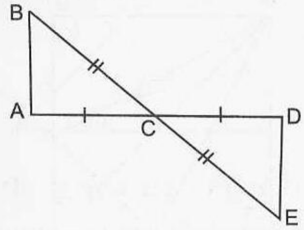
$y = 63°$

$\therefore \angle ABC = y + x$

$= 63° + 32° = 95°$

$\therefore \angle ABC = 95°$

Ex.20. In the given figure, C is the mid-point of AD as well as of BE. Prove that $\Delta ABC \cong \Delta DEC$.



Sol. In ΔABC and ΔDEC

$AC = CD$ [Given]

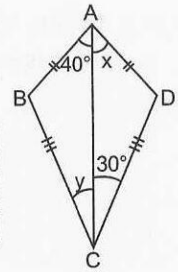
$\angle ACB = \angle DCE$ [Vertically opp. $\angle$s]

and $BC = CE$ [Given]

Since, condition of congruency (SAS) is satisfied.

$\Delta ABC \cong \Delta DEC$ **Hence proved.**

Ex.21. In the given figure, find the values of x and y.



Sol. In ΔABC and ΔADC,

$AB = AD$ [Given]

$BC = CD$ [Given]

$AC = AC$ [Common]

$\Delta ABC \cong \Delta ADC$

[$\because$ SSS-condition of congruency is satisfied]

$\therefore$ $\angle BAC = \angle CAD$

$\Rightarrow$ $x = \angle BAC = 40°$ [$CPCT$]

$\therefore$ $x = 40°$

$\angle BCA = \angle ACD$

$\Rightarrow$ $y = 30°$ [$CPCT$]

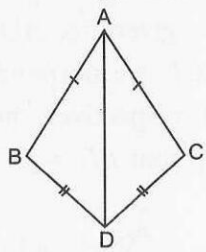
$\therefore$ $y = 30°$.

PRACTICE QUESTIONS

1. $\Delta ABC \cong \Delta PQR$, $\angle B = \angle Q = 90°$, $AC = PR$ and $AB = PQ$. The condition of congruency is

   (*a*) SAS (*b*) *AAS*

   (*c*) RHS (*d*) None of these

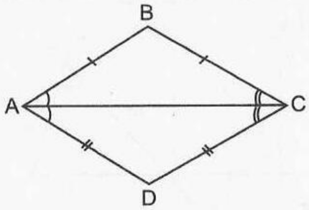
2. 

   In the given fig. above $\angle BAD = 32°$, $\angle BDC = 56°$, $\angle CAD = 2x°$, and $\angle BDA = (x + y)°$ values of x and y respectively are

   (*a*) 16°, 12° (*b*) 14°, 10°

   (*c*) 12°, 10° (*d*) None of these

3. In the given fig. AB = 4cm, CD = 3 cm, $AB = (3x - 2y)$ cm and $BC = (x + y)$ cm value of x and y respectively are

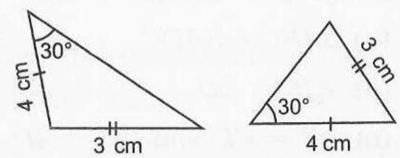
   

   (*a*) 1 cm, 3 cm (*b*) 1 cm, 2 cm

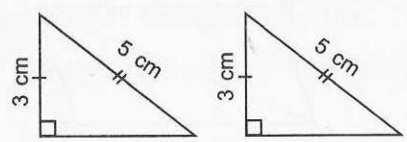
   (*c*) 2 cm 1 cm (*d*) None of these

4. State which of the pair of triangles are congruent. Mention in each case the type of congruency (namely, *SSS*, *SAS*, *AAS*, RHS)

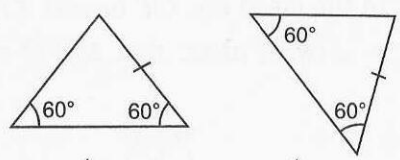
   (*a*)

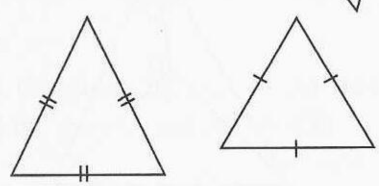
   (*b*)

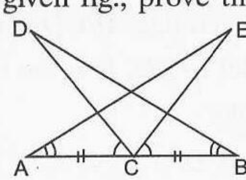
   (*c*)

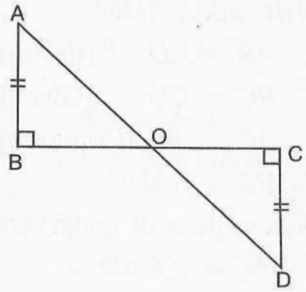
   (*d*)

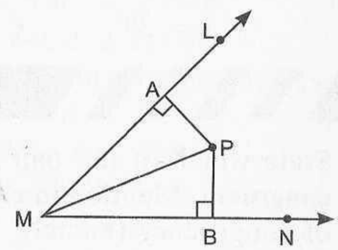
5. In the given fig., prove that $AE = BD$.

6. In the given fig., prove that AD and BC bisect each other at O.



7. In the given fig., P is a point in the interior of $\angle LMN$ such that $PA \perp ML$, $PB \perp MN$ and $PA = PB$. Show that P lies on the bisector of $\angle LMN$.

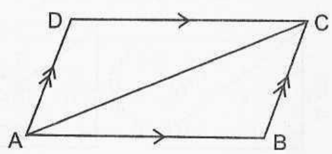


8. In the given fig., prove that

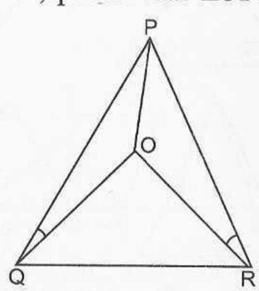
(a) $\Delta ABC \cong \Delta ADC$

(b) $\angle B = \angle D$

(c) $AB = CD$ and $BC = AD$



9. In the given fig., OP bisects $\angle P$ and $\angle OQP = \angle ORP$, prove that $\Delta OPQ \cong \Delta OPR$.

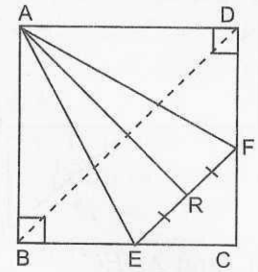


10. In the given fig., $ABCD$ is a square and EF is parallel to BD. R is the mid-point of EF. Prove that:

(a) $BE = DF$

(b) AR bisects $\angle BAD$

(c) If AR is produced it will pass through C.



11. In ΔABC, D is the mid-point of BC; AD is produced up to E so that $DE = AD$. Prove that

(a) $\Delta ABD \cong \Delta ECD$

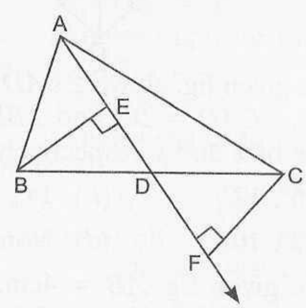
(b) $AB = EC$

(c) AB is parallel to EC.

12. Prove that the altitudes of an equilateral triangle are all equal.

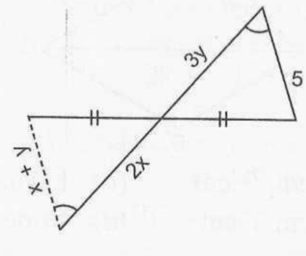
13. $ABCD$ is a trapezium, AB is parallel to DC. E is the mid-point of BC. AE is produced and meets DC produced in F. Prove that $\Delta ABE \cong \Delta FCE$.

14. In the given fig., AD is the median and BE and CF are perpendiculars drawn from B and C respectively on AD and AD produced. Prove that $BE = CF$.

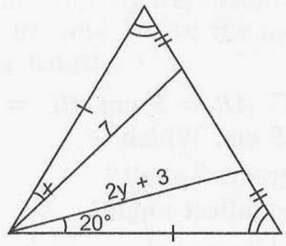


15. In the following figures, find the values of x and y.

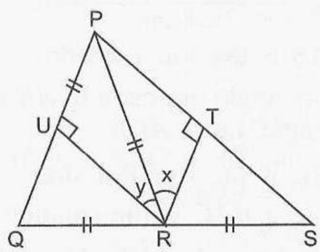
(a)



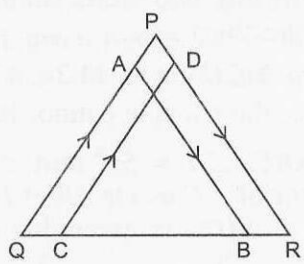
(*b*)



16. In the given fig., PQR is an equilateral triangle. Base QR is produced to S, such that $QR = RS$, $RT \perp PS$ and $RU \perp PQ$, find the value of $x + y$.



17. In the given fig., $PQ \parallel DC$, $PR \parallel AB$ and $QC = BR$. Prove that $\Delta ABQ \cong \Delta DCR$.

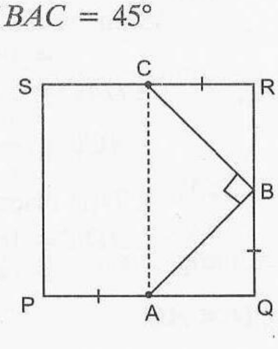


18. In the given fig., $PQRS$ is a square and A, B, C are points on the sides PQ, QR and RS respectively, such that $PA = QB = RC$ and $\angle ABC = 90°$. Prove that

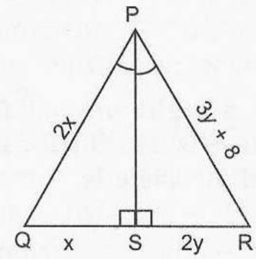
(*a*) $AQ = BR$

(*b*) $AB = BC$

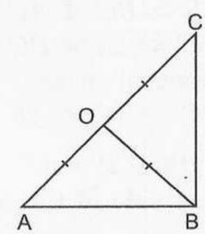
(*c*) $\angle BAC = 45°$



19. In the given fig., PS bisects $\angle P$ of ΔPQR and $PS \perp QR$. If $PQ = 2x$, $QS = x$, $PR = 3y + 8$ and $SR = 2y$, find the values of x and y.



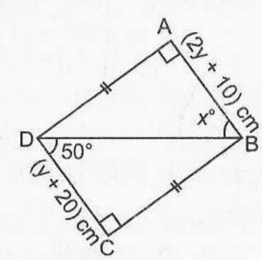
20. In the given fig., OAB is a triangle with $OA = OB$. AO is produced to C such that $OA = OC$ and CB is joined. Find $\angle ABC$.



21. Prove that the median on equal sides of an isosceles triangle are equal.

22. Prove that the bisectors of base angles of an isosceles triangle are equal.

23. In the given fig., find the values of x and y.



24. In the given fig., $\angle x = \angle y$ and $AB = CB$. Then, prove that $AE = CD$.