# New Stewart School, Cuttack

## QUESTIONS BANK CLASS 10
### Class 10 - Mathematics

**Section A**

1. The volume of a conical tent is $462\ m^3$ and the area of the base is $154\ m^2$. The height of the cone is **[1]**

a) 12 m
b) 9 m
c) 15 m
d) 24 m

[2022]

2. If the height of two cones are in the ratio of $1:4$ and the radii of their bases are in the ratio $4:1$, then the ratio of their volumes is: **[1]**

a) $2:3$
b) $3:4$
c) $4:1$
d) $1:2$

3. The radius and height of a right circular cone are in the ratio of $5:12$ and its volume is $2512\ \text{cm}^3$. The slant height of the cone is: **[1]**
(Take $\pi = 3.14$)

a) 26 cm
b) 24 cm
c) 14 cm
d) 16 cm

4. A solid metallic cone of radius 10 cm and height 12 cm is melted to form a wire of uniform cross-section $1\ \text{cm}^2$. The length of wire formed is: **[1]**

a) $400 \times \pi\text{m}$
b) $2 \times \pi\text{m}$
c) $4 \times \pi\text{m}$
d) $40 \times \pi\text{cm}$

5. Two circular cylinders of equal volumes have their heights in the ratio $1:2$. The ratio of their radii is: **[1]**

a) $\sqrt{2}:1$
b) $1:2$
c) $1:4$
d) $1:\sqrt{2}$

6. If $\sin\theta + \operatorname{cosec}\theta = 2$ , then $\sin^3\theta + \operatorname{cosec}^3\theta$ is equal to: **[1]**

a) $2\cos\theta$
b) $2\sin\theta$
c) $-2\sin\theta$
d) 2

7. $(1+\sin A)(1-\sin A)$ is equal to **[1]**

a) $\sin^2 A$
b) $\operatorname{cosec}^2 A$
c) $\cos^2 A$
d) $\sec^2 A$

[2023]

8. $(\cot \text{A} - \cot \text{B})^2 + (1 + \cot \text{A} \cot \text{B})^2$ is equal to: **[1]**

a) $\sec^2 A - \operatorname{cosec}^2 A$

b) $\operatorname{cosec}^2 A . \operatorname{cosec}^2 B$

c) $(1+\tan A)^2 - (1-\cot A)^2$

d) $\sec^2 A - \cos^2 A$

9. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$ , then $x^2+y^2$ is equal to **[1]**

a) 0

b) 1

c) $1/2$

d) $3/2$

10. $\left(1+\cot^2 A\right)+\left(1+\tan^2 A\right)$ is equal to: **[1]**

a) $\sec^2 A - \operatorname{cosec}^2 A$

b) $\frac{1}{\sin^2 A - \cos^2 A}$

c) $\sin^2 A - \sin^4 A$

d) $\sec^2 A \cdot \operatorname{cosec}^2 A$

11. The value of the expression 1 - $\frac{\sin^2 y}{1+\cos y} + \frac{1+\cos y}{\sin y}$ - $\frac{\sin y}{1-\cos y}$ is **[1]**

a) 1

b) cos y

c) sin y

d) 0

12. $\frac{1}{\cot\theta+\tan\theta}$ is equal to: **[1]**

a) $\sin\theta + \cos\theta$

b) $\sin\theta \cdot \cos\theta$

c) $\frac{1}{\sin\theta\cdot\cos\theta}$

d) $\frac{1}{\sin\theta+\cos\theta}$

13. $\sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)} = ?$ **[1]**

a) cot A

b) $\sec A \tan A$

c) $\sec A + \tan A$

d) $\sec A - \tan A$

14. A fair die is thrown once. The probability of getting an even prime number is **[1]**

a) $\frac{1}{6}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{2}{3}$

15. If three different coins are tossed together, find the probability of getting two heads: **[1]**

a) 1/2

b) 5/8

c) 3/8

d) 3/4

16. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game. **[1]**

a) $\frac{1}{4}$

b) $\frac{7}{8}$

c) $\frac{1}{2}$

d) $\frac{3}{4}$

[2019]

17. In a simultaneous throw of two dice, what is the probability of getting a total of 10 or 11? **[1]**

a) 7/12

b) 5/36

c) 1/6

d) 1/4

18. Which of the following cannot represent the probability of an event? **[1]**

a) $\left(\frac{1.8}{3}\right)$

b) $\left(\frac{0.4}{5}\right)$

c) $\left(\frac{1}{0.4}\right)$

d) $\left(\frac{2}{65}\right)$

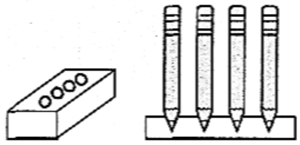
19. A die is thrown once. The probability of getting a number which has at least two factors is: **[1]**

a) 2/3
b) 5/6
c) 1
d) 1/2

20. A bag contains 3 red and 2 blue marbles. A marble is drawn at random. The probability of drawing a black marble is **[1]**

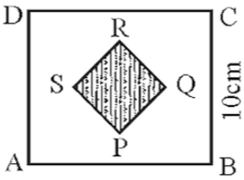
a) 0
b) $\frac{3}{5}$
c) $\frac{1}{5}$
d) $\frac{2}{5}$

21. A pen-stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm $\times$ 10 cm $\times$ 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Then, the volume of wood in the entire stand (see the figure). [Taken $\pi$ = $\frac{22}{7}$] **[1]**



a) 523.54 $cm^3$
b) 523.45 $cm^3$
c) 523 $cm^3$
d) 532.45 $cm^3$

22. A square of side 5 cm is drawn in the interior of another square of side 10 cm and shaded as shown in the figure below **[1]**




A point is selected at random from the interior of square ABCD. The probability that the point will be chosen from the shaded part is

a) 0.25
b) 0.30
c) 0.10
d) 0.05

23. Water in a canal 6 m wide and 2 m deep, is flowing with the speed of $18\ \mathrm{km/h}$. **[1]**

**Statement (1):** The volume of water that flows through the canal in 20 minutes $= 6 \times 2 \times \left(18 \times \frac{5}{18}\right) \times 20 \times 60\ \mathrm{m}^3$ .

**Statement (2):** The volume of water that flows through the canal $= 6 \times 2 \times 18 \times 20\ \mathrm{m}^3$ .

a) Statement 1 is false, and statement 2 is true.
b) Both the statements are false.
c) Statement 1 is true, and statement 2 is false.
d) Both the statements are true.

24. A solid wooden cylinder is of height h cm and radius r cm. A conical cavity of the same height and the same radius is drilled out of the solid cylinder. **[1]**

**Statement (1):** The volume of the remaining wood = volume of the solid cylinder - volume of the cone drilled.

**Statement (2):** The volume of the remaining wood $= \pi \mathrm{r}^2 \mathrm{~h} - \frac{1}{3} \pi \mathrm{r}^2 \mathrm{~h} = \frac{2}{3} \pi \mathrm{r}^2 \mathrm{~h}$

a) Both the statements are true.

b) Both the statements are false.

c) Statement 1 is true, and statement 2 is false.

d) Statement 1 is false, and statement 2 is true.

25. **Statement I:** $\frac{\sin A}{1+\cos A} = \operatorname{cosec} A - \cot A$ . **[1]**

**Statement II:** $\cos^2 \theta - 1 = \sin^2 \theta$ .

a) Both the Statements are false.

b) Statement I is false and Statement II is true.

c) Statement I is true and Statement II is false.

d) Both the Statements are true.

26. **Statement I:** An integer is chosen between 0 and 100. The probability that the number is divisible by 7 will be $\frac{14}{99}$. **[1]**

**Statement II:** An integer is chosen between 0 and 100. The probability that the number is not divisible by 7 will be $\frac{85}{99}$.

Which of the following is valid?

a) Both the Statements are true.

b) Statement I is false and Statement II is true.

c) Statement I is true and Statement II is false.

d) Both the Statements are false.

27. In a lottery ticket, there are 20 prizes and 25 blanks. **[1]**

**Statement (1):** Probability of not getting the prize $= 1 - \frac{20}{45}$

**Statement (2):** $\mathrm{P}($ getting prize$) + \mathrm{P}($ blank$) = 1$

a) Both the statements are true.

b) Statement 1 is false, and statement 2 is true.

c) Both the statements are false.

d) Statement 1 is true, and statement 2 is false.

28. **Assertion (A):** From a solid wooden cylinder of height 15 cm and diameter 14 cm, of conical cavity of same height and same base diameter is hollowed out. The volume of the cone is $770\ cm^3$. **[1]**

**Reason (R):** The volume of a cylinder of height $h$ and radius $r$ is $\pi r^2 h$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

29. $x = (\operatorname{cosec} A + \cot A)(1 - \cos A)$ **[1]**

**Assertion (A):** $x = \sin A$

**Reason (R):** $x = \left(\frac{1}{\sin A} + \frac{\cos A}{\sin A}\right)(1 - \cos A) = \frac{\sin^2 A}{\sin A} = \sin A$

a) Both A and R are true and R is the

b) Both A and R are true but R is not the correct explanation of A.

correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

30. **Assertion (A):** If $\sec\theta + \tan\theta = p$ , then $\sec\theta = \frac{2p}{p^2+1}$ **[1]**

**Reason (R):** $\sec^2\theta - \tan^2\theta = 1$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

31. **Assertion (A):** In a cricket match, a batsman hits a boundary 9 times out of 45 balls, he plays. The probability that in a given ball, he does not hit the boundary is $\frac{4}{5}$. **[1]**

**Reason** $(R):\ P(E) + P(\text{not}\, E) = 1$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

32. **Assertion (A):** Two players Sania and Asma play a tennis match. If the probability of Sania winning the match is 0.79, then the probability of Asma winning the match is 0.21. **[1]**

**Reason (R):** The sum of probabilities of two complementary events is 1.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

33. **Assertion (A):** If two coins are tossed together, then the probability of getting no tail is $\frac{1}{4}$. **[1]**

**Reason (R):** The probability $P(E)$ of an event $E$ satisfies $0 \leq P(E) \leq 1$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

**Section B**

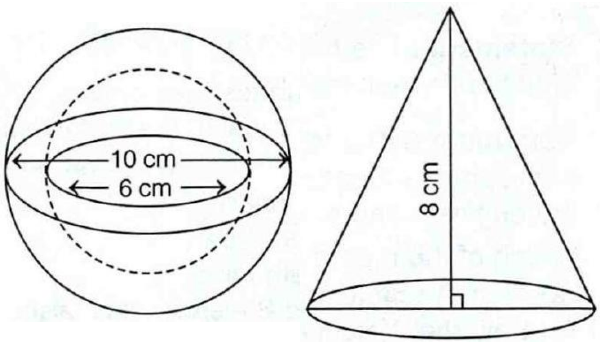
34. A rectangular water tank of base $11\ m \times 6\ m$ contains water upto a height of 5 m if the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find the height of water level in the tank. **[3]**

35. A solid metallic circular cylinder of radius 14 cm and height 12 cm is melted and recast into small cubes of edge 2 cm. How many such cubes can be made from the solid cylinder? **[3]**

36. A famous sweet shop "Madanlal Sweets" sells tinned resgullas. The tin container is cylindrical in shape with diameter 14 cm, height 16 cm, and it can hold 20 spherical rasgullas of diameter 6 cm and sweetened liquid such that the can is filled and then sealed. Find out how much sweetened liquid the can contains. [take, $\pi = 3.14$] **[3]**

37. Find the curved and total surface area of a hemisphere of radius 10 cm. [Take π = 3.14] **[3]**

38. A hollow sphere of external diameter 10 cm and internal diameter 6 cm is melted and made into a solid right circular cone of height 8 cm. Find the radius of the cone so formed. $\left[\text{use } \pi = \frac{22}{7}\right]$ **[3]**




[2025]

39. A solid cone of radius 5 cm and height 9 cm is melted and made into small cylinders of radius of 0.5 cm and height 1.5 cm. Find the number of cylinders so formed. **[3]**

[2022]

40. The surface area of a solid sphere is $1256\ cm^2$. It is cut into two hemispheres. Find the total surface area and the volume of a hemisphere. Take $\pi = 3.14$ **[3]**

41. Prove that $(\sin\theta + \cos\theta + 1)\ (\sin\theta - 1 + \cos\theta)\sec\theta \operatorname{cosec}\theta = 2$ **[3]**

42. Prove that: **[3]**
$\frac{\cot^2 A}{\operatorname{cosec} A - 1} - 1 = \operatorname{cosec} A$

43. Prove that: $\cot A - \tan A = \frac{2\cos^2 A - 1}{\sin A \cos A}$ **[3]**

44. Prove the following identity: **[3]**

$\sec^2 A + \operatorname{cosec}^2 A = \sec^2 A \operatorname{cosec}^2 A$

45. Prove the following identity: **[3]**
$\sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)} = \sec A - \tan A$

46. Prove the following identity: **[3]**
$\left(1 + \tan^2 A\right) + \left(1 + \cot^2 A\right) = \left(\frac{1}{\sin^2 A - \sin^4 A}\right)$

47. Prove that $\frac{\sec A - 1}{\sec A + 1} = \frac{1 - \cos A}{1 + \cos A}$. **[3]**

[2007]

48. A bag contains 5 white, 2 red and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn is a red ball? **[3]**

[2022]

49. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, find the number of blue balls in the bag. **[3]**

50. A letter is chosen from the word 'TRIANGLE'. What is the probability that it is a vowel? **[3]**

51. Sarita and Hemlata are friends. What is the probability that both will have **[3]**
    i. the same birthday?
    ii. different birthdays? (ignoring a leap year)

**Section C**

52. A metallic cone of base radius 8 cm and height 6.912 cm is melted down and shaped into a sphere without any loss of metal. Find the radius of the sphere. [Take $\pi$ = 3.14] **[4]**

53. Water is flowing at the rate of 8 m per second through a circular pipe whose internal diameter is 2 cm, into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the water level in 30 minutes. **[4]**

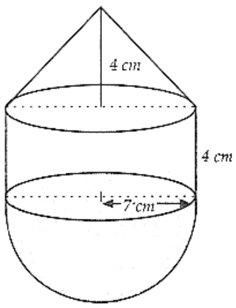
54. A circus tent is in the shape of a cylinder surmounted by a cone. The diameter of the cylindrical portion is 24 m and its height is 11 m. If the vertex of the cone is 16 m above the ground, find the area of the canvas used to make the tent. **[4]**

55. From a solid wooden cylinder of height 28 cm and diameter 6 cm, two conical cavities are hollowed out. The diameters of the cones are also of 6 cm and height 10.5 cm. Taking $\pi = \frac{22}{7}$ find the volume of the remaining solid. **[4]**

[2020]

56. A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 7 m each and the total height of the tent is 14 m. Find the **[4]**
   i. quantity of air contained inside the tent.
   ii. radius of a sphere whose volume is equal to the quantity of air inside the tent.
   $\left[\text{use } \pi = \frac{22}{7}\right]$

57. The following figure represents a solid consisting of a right circular cylinder with a hemisphere at one end and a cone at the other. Their common radius is 7 cm. The height of the cylinder and cone are each of 4 cm. Find volume of the solid. **[4]**




[2018]

58. A solid metallic cone, with radius 6 cm and height 10cm, is made of some heavy metal A. In order to reduce its weight, a conical hole is made in the cone as shown and it is completely filled with a lighter metal B. The conical hole has a diameter of cm and depth 4 cm calculate the ratio of the volume of metal A to the volume of the metal B in the solid. **[4]**

59. If $x = r\cos A\cos B, y = r\cos A\sin B$ and z $= r\sin A$, prove that $x^2 + y^2 + z^2 = r^2$ **[4]**

60. If 1 + $\sin^2\theta$ = 3 sin $\theta$ cos $\theta$, then prove that tan $\theta$ = 1 or $\frac{1}{2}$. **[4]**

61. If tan $\theta$ = $\frac{20}{21}$, then prove that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$ **[4]**

62. Prove the following identities. **[4]**
   i. (sec A - sin A) (cosec A + cos A) = $\sin^2$ A tan A + cot A
   ii. (1 + cot A + tan A) (sin A - cos A) = $\frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A}$

63. Prove that: $\frac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta} = \sec^2\theta + \tan^2\theta$ **[4]**

[2022]

64. Prove: $(\cot A - \operatorname{cosec} A)^2 = \frac{1-\cos A}{1+\cos A}$ **[4]**

65. Prove that: (1 + cot $\theta$ - cosec $\theta$) (1 + tan $\theta$ + sec $\theta$) = 2 **[4]**

[2018]

66. Ramesh chooses a date at random in January for a party. **[4]**

| January | | | | | |
|---|---|---|---|---|---|
| Mon | | 6 | 13 | 20 | 27 |

| | | | | | |
|---|---|---|---|---|---|
| Tue | | 7 | 14 | 21 | 28 |
| Wed | 1 | 8 | 15 | 22 | 29 |
| Thurs | 2 | 9 | 16 | 23 | 30 |
| Fri | 3 | 10 | 17 | 24 | 31 |
| Sat | 4 | 11 | 18 | 25 | |
| Sun | 5 | 12 | 19 | 26 | |

Find the probability that he chooses:

i. a Wednesday.
ii. a Friday.
iii. a Tuesday or a Saturday.

67. Find the probability that a number selected at random from the numbers 1, 2, 3, ..., 35 is a **[4]**

i. prime number
ii. multiple of 7
iii. multiple of 3 or 5

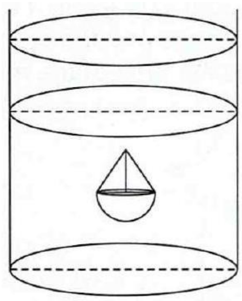
68. Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more than 9. **[4]**

69. Two players Niharika and Shreya play a tennis match. It is known that the probability of Niharika winning the match is 0.62. What is the probability of Shreya winning the match? **[4]**

**Section D**

70. A rocket is in the form of a right circular cylinder closed at the lower end and surmounted by a cone with the same radius as that of the cylinder. The diameter and height of the cylinder are 6 cm and 12 cm, respectively. If the slant height of the conical portion is 5 cm, then find the total surface area and volume of the rocket. [take $\pi$ = 3.14] **[5]**

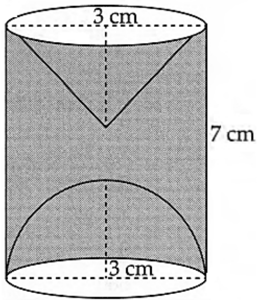
71. A solid is in the shape of a hemisphere of radius 7 cm, surmounted by a cone of height 4 cm. The solid is immersed completely in a cylindrical container filled with water to a certain height. If the radius of the cylinder is 14 cm, find the rise in the water level. **[5]**



[2023]

72. A hemispherical and a conical hole is scooped out of a solid wooden cylinder. Find the volume of the remaining solid where the measurements are as follows: **[5]**
The height of the solid cylinder is 7 cm, radius of each of hemisphere, cone and cylinder is 3 cm.

Height of cone is 3 cm. Give your answer correct to the nearest whole number. $\left[\text{Take } \pi = \frac{22}{7}\right]$




[2019]

73. From a solid cylinder whose height is 16 cm and radius is 12 cm, a conical cavity of height 8 cm and of base radius 6 cm is hollowed out. Find the volume and total surface area of the remaining solid. **[5]**

74. Prove that: $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)$ sin$^2\,\theta$ cos$^2\theta$ = $\frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$ **[5]**

75. If $\sqrt{3}$ cot$^2\,\theta$ - 4 cot $\theta$ + $\sqrt{3}$ = 0, then find the value of tan$^2\,\theta$ + cot$^2\,\theta$. **[5]**

**Solution**
**QUESTIONS BANK CLASS 10**
**Class 10 - Mathematics**
**Section A**

1.
**(b)** 9 m
**Explanation:**
Given, volume of the conical tent $= 462\ m^3$
and area of the base $= 154\ m^2$
$\therefore$ Volume of the conical tent
$= \frac{1}{3} \times$ (Area of the base) $\times$ Height
$\Rightarrow 462 = \frac{1}{3} \times 154 \times$ Height
$\Rightarrow$ Height $= \frac{462 \times 3}{154}$
$= 3 \times 3 = 9\ m$

2.
**(c)** $4:1$
**Explanation:**
Let height of cones be 1a and 4a and radius of the cones be 4 b and 1b.
Volume of cone $= \frac{1}{3}\pi r^2 h$
Volume of 1st cone, $V = \frac{1}{3}\pi(4\ b)^2 1a = \frac{1}{3}\pi \times 16\ b^2 \times a$
Volume of 2nd cone, $v = \frac{1}{3}\pi(1\ b)^2 4a = \frac{1}{3}\pi \times b^2 \times 4a$
$\Rightarrow \frac{V}{v} = \frac{\frac{1}{3}\pi \times 16\ b^2 \times a}{\frac{1}{3}\pi \times b^2 \times 4a}$
$= \frac{16}{4}$
$= \frac{4}{1}$.
$= 4:1$

3. **(a)** 26 cm
**Explanation:**
Given, radius$(r)$: height $(h) = 5:12$
Let $r = 5x$ and $h = 12x$
Volume of cone $= \frac{1}{3}\pi r^2 h$
$\Rightarrow 2512 = \frac{1}{3} \times 3.14 \times (5x)^2 \times 12x$
$\Rightarrow 2512 = 3.14 \times 25x^2 \times 4x$
$\Rightarrow x^3 = \frac{2512}{3.14 \times 25 \times 4}$
$\Rightarrow x^3 = \frac{2512}{314}$
$\Rightarrow x^3 = 8$
$\Rightarrow x = \sqrt[3]{8}$
$\Rightarrow x = 2\ cm$
$\Rightarrow r = 5x = 5 \times 2 = 10\ cm$

$\Rightarrow\ h = 12x = 12 \times 2 = 24\ cm$

Curved surface area $= \pi rl$

$l^2 = r^2 + h^2$

$\Rightarrow l^2 = 10^2 + 24^2$

$\Rightarrow l^2 = 100 + 576$

$\Rightarrow l^2 = 676$

$\Rightarrow l = \sqrt{676} = 26\ cm$

4.

**(c)** $4 \times \pi$m

**Explanation:**

Given,

A solid metallic cone of radius (r) 10 cm and height (h) 12 cm is melted to form a wire of uniform cross-section $1\ cm^2$.

Let length of wire formed be l cm.

$\therefore$ Volume of cone = Volume of wire

$\Rightarrow \frac{1}{3}\pi r^2 h =$ Area of cross-section $\times$ length

$\Rightarrow \frac{1}{3}\pi \times 10^2 \times 12 = 1 \times l$

$\Rightarrow l = 4 \times \pi \times 10^2$

$\Rightarrow l = 400\pi\ cm$

$\Rightarrow l = \frac{400\pi}{100}\ m$

$\Rightarrow l = 4 \times \pi m$

5. **(a)** $\sqrt{2} : 1$

**Explanation:**

Let radius and heights of two cylinders be $r, h$ and $R, H$.

Given,

$\frac{h}{H} = \frac{1}{2}$

Volume of cylinder $1 = v$

Volume of cylinder $2 = V$

$\Rightarrow v = V$

$\Rightarrow \pi r^2\ h = \pi R^2 H$

Divide by $\pi$ on both sides, we get:

$\Rightarrow r^2 = R^2 \times \frac{H}{h}$

$\Rightarrow \frac{r^2}{R^2} = \frac{2}{1}$

$\Rightarrow \left(\frac{r}{R}\right)^2 = \frac{2}{1}$

$\Rightarrow \frac{r}{R} = \sqrt{\frac{2}{1}}$

$\Rightarrow \frac{r}{R} = \frac{\sqrt{2}}{1}$

$\therefore r : R = \sqrt{2} : 1$

6.

**(d)** 2

**Explanation:**

$\sin\theta + \text{cosec}\theta = 2$

Let,

$\Rightarrow \sin\theta = x$

$\Rightarrow \text{cosec}\theta = \frac{1}{x}$

$\Rightarrow x + \frac{1}{x} = 2$

$\Rightarrow \frac{x^2+1}{x} = 2$

$\Rightarrow x^2 + 1 = 2x$

$\Rightarrow x^2 + 1 - 2x = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$

Hence,

$\sin\theta = 1$ and $\text{cosec}\theta = 1$

$\Rightarrow \sin^3\theta + \text{cosec}^3\theta$

$\Rightarrow 1^3 + 1^3$

$\Rightarrow 2$

7.

**(c)** $\cos^2 A$

**Explanation:**

We have, $(1 + \sin A)(1 - \sin A)$

$= 1^2 - \sin^2 A\left[\because a^2 - b^2 = (a+b)(a-b)\right]$

$= 1 - \sin^2 A = \cos^2 A\left[\because 1 - \sin^2\theta = \cos^2\theta\right]$

8.

**(b)** $\text{cosec}^2 A.\ \text{cosec}^2 B$

**Explanation:**

Solving,

$\Rightarrow (\cot A - \cot B)^2 + (1 + \cot A \cot B)^2$

$\Rightarrow \cot^2 A + \cot^2 B - 2\cot A\cot B + 1 + \cot^2 A.\cot^2 B + 2\cot A\cot$ B

$\Rightarrow \cot^2 A + \cot^2 A.\cot^2 B + 1 + \cot^2 B$

$\Rightarrow \cot^2 A\left(1 + \cot^2 B\right) + \left(1 + \cot^2 B\right)$

$\Rightarrow \left(1 + \cot^2 B\right)\left(1 + \cot^2 A\right)$

$\Rightarrow \left(1 + \frac{\cos^2 B}{\sin^2 B}\right)\left(1 + \frac{\cos^2 A}{\sin^2 A}\right)$

$\Rightarrow \left(\frac{\sin^2 B + \cos^2 B}{\sin^2 B}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin^2 A}\right)$

Substituting, $\sin^2\theta + \cos^2\theta = 1$, we get:

$\Rightarrow \left(\frac{1}{\sin^2 B}\right)\left(\frac{1}{\sin^2 A}\right)$

$\Rightarrow \text{cosec}^2 B.\ \text{cosec}^2 A$

9.

**(b)** 1

**Explanation:**

We have,

$x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta\left(\sin^2\theta\right) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta$ [ $\because y\cos\theta = x\sin\theta$]

$\Rightarrow x\sin\theta\left(\sin^2\theta + \cos^2\theta\right) = \sin\theta\cos\theta$

$\Rightarrow x\sin\theta = \sin\theta\cos\theta \left[\because \sin^2A + \cos^2A = 1\right]$

$\Rightarrow x = \cos\theta$

Now, $x\sin\theta = y\cos\theta$

$\Rightarrow \cos\theta\sin\theta = y\cos\theta$

$\Rightarrow y = \sin\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1$

10.

**(d)** $\sec^2A \cdot \mathrm{cosec}^2A$

**Explanation:**

Solving,

$\Rightarrow \left(1 + \cot^2 A\right) + \left(1 + \tan^2 A\right)$

$\Rightarrow \mathrm{cosec}^2 A + \sec^2 A$

$\Rightarrow \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A}$

$\Rightarrow \frac{\cos^2 A+\sin^2 A}{\sin^2 A\cdot\cos^2 A}$

Substituting, $\sin^2 A + \cos^2 A = 1$, we get:

$\Rightarrow \frac{1}{\sin^2 A\cdot\cos^2 A}$

$\Rightarrow \mathrm{cosec}^2 A \cdot \sec^2 A$

11.

**(b)** cos y

**Explanation:**

Given expression can be written as

$\frac{1+\cos y-\sin^2 y}{1+\cos y} + \frac{\left(1-\cos^2 y\right)-\sin^2 y}{\sin y(1-\cos y)}$

$= \frac{\cos y(1+\cos y)}{1+\cos y} + \frac{1-1}{\sin y(1-\cos y)}$

= cos y + 0 = cos y

12.

**(b)** $\sin\theta \cdot \cos\theta$

**Explanation:**

Solving,

$\Rightarrow \frac{1}{\cot\theta+\tan\theta}$

$\Rightarrow \frac{1}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \dfrac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos\theta\cdot\sin\theta}}$

$\Rightarrow \dfrac{\cos\theta\cdot\sin\theta}{\cos^2\theta+\sin^2\theta}$

Substituting, $\sin^2\theta + \cos^2\theta = 1$, we get:

$\Rightarrow \sin\theta\cdot\cos\theta$

13.

**(d)** $\sec A - \tan A$

**Explanation:**

Rationalize the expression,

$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)}$

$\Rightarrow \sqrt{\left(\frac{1-\sin A}{1+\sin A}\right)\times\frac{1-\sin A}{1-\sin A}}$

$\Rightarrow \sqrt{\left(\frac{(1-\sin A)^2}{1-\sin^2 A}\right)}$

$\Rightarrow \sqrt{\left(\frac{(1-\sin A)^2}{\cos^2 A}\right)}$

$\Rightarrow \dfrac{(1-\sin A)}{\cos A}$

$\Rightarrow \dfrac{1}{\cos A}-\dfrac{\sin A}{\cos A}$

$\Rightarrow \sec A - \tan A$

14. **(a)** $\frac{1}{6}$

**Explanation:**

When a die is thrown, the sample space is,

$S = \{1, 2, 3, 4, 5, 6\}$.

Let A be the event of getting an even prime number,

$\therefore A = \{2\}$.

Hence, the no. of favourable outcomes to $A = 1$.

$\therefore P(A) = \frac{1}{6}$.

15.

**(c)** 3/8

**Explanation:**

When three coins are tossed together, the possible outcomes are: {(H, H, H), (H, H, T), (H, T, H), (T, H, H), (H, T, T), (T, H, T), (T, T, H), (T, T, T)}

Total number of outcomes $= 8$

Let E be the event of getting exactly two heads,

$E = \{(H, H, T), (H, T, H), (T, H, H)\}$

The number of favorable outcomes to the event $E = 3$

$\therefore P(E) = \dfrac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \dfrac{3}{8}$

16.

**(d)** $\frac{3}{4}$

**Explanation:**

When a coin is thrown three times

Then sample space $\mathrm{S} = \{\mathrm{HHH}$, HHT, HTH, THH, HTT, THT, TTH, TTT $\}$

$\therefore n(\mathrm{S}) = 8$

Let $E$ : event of getting same result in all the three tosses is considered as success $= \{$HHH, TTT $\}$

$\therefore n(\mathrm{E}) = 2$

Thus P (prob. of success) $= \mathrm{P(E)} = \frac{n(\mathrm{E})}{n(\mathrm{S})} = \frac{2}{8} = \frac{1}{4}$

$\therefore$ required prob. of losing the game $= 1 - \mathrm{P}$ (success) $= 1 - \frac{1}{4} = \frac{3}{4}$

17.

**(b)** 5/36

**Explanation:**

When two dice are thrown simultaneously, each die has 6 possible outcomes.

Total number of outcomes $= 36$

Let E be the event of getting a total of 10 or 11,

$E = \{(4, 6), (5, 5), (6, 4), (5, 6), (6, 5)\}$

The number of favorable outcomes to the event $E = 5$

$\therefore P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{5}{36}$

18.

**(c)** $(\frac{1}{0.4})$

**Explanation:**

We know that,

$0 \leq P(E) \leq 1$

$\left(\frac{1}{0.4}\right) = 2.5$

Since this value is greater than 1, it cannot be a probability.

19.

**(b)** 5/6

**Explanation:**

When a die is thrown, there are 6 possible outcomes:

$S = \{1, 2, 3, 4, 5, 6\}$

Total number of outcomes $= 6$

Let E be the event of getting a number with at least two factors, then

$E = \{2, 3, 4, 5, 6\}$

The number of favorable outcomes to the event $E = 5$

$\therefore P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{5}{6}$

20. **(a)** 0

**Explanation:**

Given, number of red marbles $n$ (Red) $= 3$,

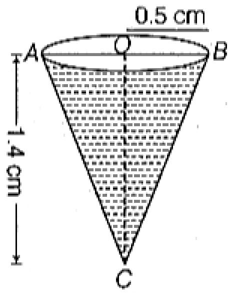
number of blue marbles, $n$ (Blue) $= 2$

and number of black marbles $n$ (Black) $= 0$

Total number of marbles

= $n$( Red ) + $n$( Blue ) + $n$( Black )
= 3 + 2 + 0 = 5
$\therefore$ The probability of drawing a black marble
$= \dfrac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \dfrac{0}{5} = 0$

21. **(a)** 523.54 $cm^3$
**Explanation:**
Given, length of cuboid, l = 15 cm,
breadth of cuboid, b = 10 cm
and height of cuboid, h = 3.5 cm
$\therefore$ Volume of the cuboid = l $\times$ b $\times$ h
= 15 $\times$ 10 $\times$ 3.5 = 525 $cm^3$
Also, radius of a conical depression, r = 0.5 cm
nd height of a conical depression, h = 1.4 cm
$\therefore$ Volume of one conical depression = $\frac{1}{3}\pi r^2 h$




$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$
$= \frac{22}{3} \times \frac{1}{2} \times \frac{1}{2} \times \frac{2}{10} = \frac{11}{30}$ $cm^3$
Now, volume of 4 conical depressions
= 4 $\times$ Volume of one conical depression = $4 \times \frac{11}{30} = \frac{22}{15}$ $cm^3$
$\therefore$ Volume of wood in the pen-stand = Volume of the cuboid - Volume of 4 conical depressions
$= 525 - \frac{22}{15} = 525 - 1.46$
= 523.54 $cm^3$

22. **(a)** 0.25
**Explanation:**
Area of the square ABCD = $(BC)^2 = 10^2 = 100$ $cm^2$
and area of the square POPS = $(\text{Side})^2 = (5)^2$ [$\because$ side = 5 cm, given]
= 25 $cm^2$
$\therefore$ P (that the point will be chosen from the shaded part)
$= \dfrac{\text{Area of the square } PQRS}{\text{Area of the square } ABCD} = \dfrac{25}{100} = 0.25$

23.
**(c)** Statement 1 is true, and statement 2 is false.
**Explanation:**
Given, water in a canal 6 m wide and 2 m deep, is flowing with the speed of $18\ km/h$.
Volume of water flowing through the canal = Area of cross-section $x$ speed $x$ time
$\text{Area of cross-section} = \text{width} \times \text{depth} = (6 \times 2)m^2$

$\text{Speed} = 18\ km/h = 18 \times \frac{1000}{3600} = \left(18 \times \frac{5}{18}\right) m/sec$

$\text{Time} = 20 \text{ minutes} = 20 \times 60 sec$

$\therefore$ Volume $= 6 \times 2 \times 18 \times \frac{5}{18} \times 20 \times 60\ m^3$

Thus, Statement 1 is true, and statement 2 is false.

24. **(a)** Both the statements are true.

**Explanation:**

The volume of the remaining wood is the volume of the original solid cylinder minus the volume of the conical cavity drilled out.

Let radius and height of cylinder and conical cavity (both are equal) be r and h units.

Volume of the cylinder: $\pi r^2 h$

Volume of the cone: $\frac{1}{3}\pi r^2\ h$

The volume of the remaining wood = Volume of the solid cylinder - Volume of the cone drilled.

$= \pi r^2 h - \frac{1}{3}\pi r^2 h$

$= \left(1 - \frac{1}{3}\right)\pi r^2 h$

$= \left(\frac{3-1}{3}\right)\pi r^2 h$

$= \frac{2}{3}\pi r^2 h.$

$\therefore$ Both the statements are true.

25.

**(c)** Statement I is true and Statement II is false.

**Explanation:**

**Statement I:** $\frac{\sin A}{1+\cos A} = \operatorname{cosec}A - \cot A$

$LHS = \frac{\sin A}{1+\cos A}$

$= \frac{\sin A}{1+\cos A} \times \frac{1-\cos A}{1-\cos A}$

[multiplying numerator and denominator by $(1 - \cos A)$]

$= \frac{\sin A(1-\cos A)}{\left(1^2-\cos^2 A\right)} \left[\because (a-b)(a+b) = a^2 - b^2\right]$

$= \frac{\sin A(1-\cos A)}{\sin^2 A}$ [using identity, $\sin^2\theta = 1 - \cos^2\theta$]

$= \frac{1-\cos A}{\sin A} = \frac{1}{\sin A} - \frac{\cos A}{\sin A}$

$= \operatorname{cosec}A - \cot A = RHS \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \text{ and } \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$

Hence, Statement I is true.

**Statement II:** We have, $\cos^2\theta - 1 = \sin^2\theta$

We know that $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta - 1 = -\sin^2\theta$

Hence, Statement II is false.

26. **(a)** Both the Statements are true.

**Explanation:**

**Statement I:** The total number of integers between 0 and 100 is 99.

So, total number of possible outcomes $= 99.$

Let $E =$ Event of choosing an integer which is divisible by 7.

$= \{7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98\}$

Then, number of outcomes favourable to $E = 14$

$\therefore P(E) = \frac{14}{99}$

$\therefore$ Statement I is true.

**Statement II**

$\because P(E) + P(\bar{E}) = 1$

$\Rightarrow \frac{14}{99} + P(\bar{E}) = 1 \Rightarrow P(\bar{E}) = 1 - \frac{14}{99} = \frac{85}{99}$

$\therefore$ Statement II is true.

27. **(a)** Both the statements are true.

**Explanation:**

Given, tickets = 20 prizes and 25 blanks

total tickets $= 20 + 25 = 45$

By formula; the probability of an event = Number of favourable outcomes

Total number of possible outcomes

$P($ getting prize $) = \frac{20}{45}$

$P($ blank $) = \frac{25}{45}$

$P($ getting prize $) + P($ blank $) = \frac{20}{45} + \frac{25}{45}$

$= \frac{20+25}{45}$

$= \frac{45}{45}$

$= 1.$

Thus, $P($ getting prize $) + P($ blank $) = 1$

So, statement 2 is true.

$\Rightarrow P(\text{ blank }) = 1 - P(\text{ getting prize })$

$\Rightarrow P(\text{ blank }) = 1 - \frac{20}{45}$

We know that,

$P($ not getting the prize $) = P($blank$)$

So, statement 1 is true.

$\therefore$ Both the statements are true.

28.

**(d)** A is false but R is true.

**Explanation:**

Given,

Height of cylinder, $(H) = 15\ cm$

Diameter of cylinder, $(D) = 14\ cm$

Radius of cylinder, $r = \frac{D}{2} = \frac{14}{2} = 7\ cm$

Height of cone,(h) $= 15\ cm$

Radius of cone, (r) $= 7\ cm$

The volume of the cone is given by the formula $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 15$

$= \frac{1}{3} \times 22 \times 7 \times 15$

$= 22 \times 7 \times 3$

$= 462$

So, assertion (A) is false.

The volume of a cylinder of height $h$ and radius $r$ is given by the formula; $\pi r^2\ h$.

So, reason (R) is true.

Thus, Assertion (A) is false, but Reason (R) is true.

29. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:**

Given, $x = (\text{cosec}A + \cot A)(1 - \cos A)$

$\Rightarrow x = \left(\frac{1}{\sin\ A} + \frac{\cos A}{\sin\ A}\right)(1 - \cos A)$

$= \left(\frac{1+\cos A}{\sin\ A}\right)(1 - \cos A)$

$= \frac{(1+\cos A)(1-\cos A)}{\sin A}$

$= \left(\frac{1-\cos^2 A}{\sin\ A}\right)$

$= \left(\frac{\sin^2 A}{\sin\ A}\right)$

$= \sin A$

$\therefore$ Both A and R are true and R is correct reason for A.

30.

**(d)** A is false but R is true.

**Explanation:**

Given,

$\sec\theta + \tan\theta = p$

We know that,

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$(p)(\sec\theta - \tan\theta) = 1$

$\sec\theta - \tan\theta = \frac{1}{p}$

Add $\sec\theta + \tan\theta$ and $\sec\theta - \tan\theta$,

$\sec\theta + \tan\theta + \sec\theta - \tan\theta = p + \frac{1}{p}$

$2\sec\theta = \frac{p^2+1}{p}$

$\sec\theta = \frac{p^2+1}{2p}$

Assertion (A) is false.

$\sec^2\theta - \tan^2\theta = 1$ is a standard Pythagorean Identity.

Reason is true.

A is false, R is true

31. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:**

Given, a batsman hits a boundary 9 times out of 45 balls, he plays.

$P(E) = P$ (hits a target) $= \frac{9}{45} = \frac{1}{5}$

Since, $P(E) + P(\bar{E}) = 1$

$\therefore P(\bar{E}) = 1 - P(E)$

$= 1 - \frac{1}{5} = \frac{4}{5}$

$\therefore$ Probability of not hitting the boundary is $\frac{4}{5}$.

Hence, both $A$ and $R$ are true and $R$ is the correct reason for A.

32. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:**

Given, the probability of Sania winning the match $= 0.79$.

As we know that the sum of probabilities of two complementary events is 1.

$\Rightarrow P(\text{Sania wins}) + P(\text{Asma wins}) = 1$

$\therefore$ Reason (R) is true.

$\Rightarrow 0.79 + P(\text{Asma wins}) = 1$

$\Rightarrow P(\text{Asma wins}) = 1 - P(\text{ Sania wins })$

$\Rightarrow P(\text{sma wins}) = 1 - 0.79$

$\Rightarrow P(\text{Asma wins}) = 0.21$

$\therefore$ Assertion (A) is true.

$\therefore$ Both A and R are true and R is the correct explanation of A.

33.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:**

$S = \{HH, HT, TH, TT\}$

Favourable outcomes $= \{HH\}$

Total number of outcomes $= 4$

$\therefore$ Required probability

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{1}{4}$

$\therefore A$ is true.

Clearly, R is true.

Hence, both $A$ and $R$ are true and $R$ is incorrect reason for A.

## Section B

34. Base of water tank $= 11\ m \times 6\ m$

Height of water level in rectangular water tank $= 5\ m$

Volume of water in tank $= lbh = 11\ m \times 6\ m \times 5\ m = 330\ m^3$.

Let water come upto height H in cylindrical tank.

Radius $= 3.5\ cm$

Volume of cylindrical tank $= \pi r^2 H$.

$\therefore \pi r^2 H = 330$

$\Rightarrow \frac{22}{7} \times 3.5^2 \times H = 330$

$\Rightarrow \frac{22 \times 12.25 \times H}{7} = 330$

$\Rightarrow H = \frac{330 \times 7}{22 \times 12.25}$

$\Rightarrow H = \frac{2310}{269.5}$

$\Rightarrow H = \frac{60}{7} = 8\frac{4}{7}\ m$

Hence, the height of water level in cylindrical tank $= 8\frac{4}{7}\ m$.

35. Radius of a solid cylinder $(r) = 14\ cm$,

Height $(h) = 12\ cm$.

Edge of cube $(a) = 2\ cm$.

Let the no. of small cubes formed be $n$.

Volume of cylinder $= n \times$ Volume of each cubes.

$\therefore \pi r^2 h = n \times (a)^3$

$\Rightarrow \frac{22}{7} \times (14)^2 \times 12 = n \times (2)^3$

$\Rightarrow \frac{22}{7} \times 14 \times 14 \times 12 = 8n$

$\Rightarrow 22 \times 2 \times 14 \times 12 = 8n$

$\Rightarrow 8n = 7392$

$\Rightarrow n = \frac{7392}{8}$

$\Rightarrow n = 924.$

Hence, the number of cubes formed are 924.

36. Given, diameter of container $= 14\ cm$

$\therefore$ Radius of container $(R) = \frac{14}{2} = 7\ cm$ and height of container $(H) = 16\ cm$

Diameter of rasgulla $= 6\ cm$

$\therefore$ Radius of rasgulla $(r) = \frac{6}{2} = 3\ cm$

$\therefore$ Volume of container

$= \text{Volume of rasgullas} + \text{Volume of liquid}$

$\Rightarrow \pi R^2 H = 20 \times \frac{4}{3}\pi r^3 + \text{Volume of liquid}$

$\text{Volume of liquid} = \pi R^2 H - \frac{80}{3}\pi r^3$

$= \pi\left(R^2 H - \frac{80}{3}r^3\right)$

$= 3.14\left(7^2 \times 16 - \frac{80}{3} \times 3^3\right)$

$= 3.14 \times (784 - 720)$

$= 3.14 \times 64 = 200.96\ cm^3$

37. Given, radius of a hemisphere, r = 10 cm

Then, curved surface area of a hemisphere

$= 2\pi r^2 = 2 \times 3.14 \times (10)^2 = 6.28 \times 100 = 628\ cm^2$

= and total surface area of a hemisphere = $3\pi r^2$

$= 3 \times 3.14 \times (10)^2 = 9.42 \times 100 = 942\ cm^2$

38. Given, external diameter of hollow sphere $= 10\ cm$

$\therefore$ External radius $\left(r_1\right) = \frac{10}{2} = 5\ cm$

and internal diameter of hollow sphere $= 6\ cm$

$\therefore$ Internal radius $\left(r_2\right) = \frac{6}{2} = 3\ cm$

$\therefore$ Volume of hollow sphere $= \frac{4}{3}\pi\left(r_1^3 - r_2^3\right)$

$= \frac{4}{3}\pi\left[5^3 - 3^3\right]$

$= \frac{4}{3}\pi[125 - 27]$

$= \frac{392\pi}{3}\ cm^3$ ...(i)

Let $r$ be the radius and $h$ be the height of the cone.

$\because$ The hollow sphere recasted into a cone.

$\because$ Volume of cone = Volume of hollow sphere.

$\therefore \frac{1}{3}\pi r^2 h = \frac{392\pi}{3}$ [from Eq. (i)]

$\Rightarrow \frac{1}{3} \times \pi \times r^2 \times 8 = \frac{392\pi}{3}$ [$\because$ h = 8 cm]

$\Rightarrow r^2 = \frac{392}{8} = 49$

$\therefore r = 7$ [taking positive square root]

Hence, the radius of the cone is 7 cm.

39. Given, radius of a cone $(R) = 5\ cm$

Height $(H) = 9\ cm$

and radius of the cylinder $(r) = 0.5\ cm$

Height $(h) = 1.5\ cm$

$\therefore$ Number of cylinder $= \dfrac{\text{Volume of a cone}}{\text{Volume of a cylinder}}$

$= \dfrac{\frac{1}{3}\pi R^2 H}{\pi r^2 h} = \dfrac{R^2 H}{3r^2 h}$

$= \dfrac{(5)^2 \times 9}{3 \times (0.5)^2 \times 1.5} = 200$

40. Given, surface area of the sphere $= 1256\ cm^2$.

We know that, surface area of sphere $= 4\pi r^2$.

$\therefore 4\pi r^2 = 1256$

$\Rightarrow 4 \times 3.14 \times r^2 = 1256$

$\Rightarrow r^2 = \dfrac{1256}{3.14 \times 4}$

$\Rightarrow r^2 = \dfrac{1256}{12.56}$

$\Rightarrow r^2 = 100$

$\Rightarrow r = \sqrt{100}$

$\Rightarrow r = 10\ cm$

Total surface area of hemisphere $= 3\pi r^2$.

Putting values we get,

Total surface area of hemisphere $= 3 \times 3.14 \times (10)^2$

$= 3 \times 3.14 \times 100$

$= 942\ cm^2$.

Volume of hemisphere $= \dfrac{2}{3}\pi r^3$.

Volume of hemisphere $= \dfrac{2}{3} \times 3.14 \times 10^3$

$= \dfrac{2}{3} \times 3.14 \times 1000$

$= \dfrac{6280}{3}$

$= 2093\dfrac{1}{3}\ cm^3$.

Hence, the surface area of hemisphere $= 942\ cm^2$ and volume of hemisphere $= 2093\dfrac{1}{3}\ cm^3$.

41. $LHS = (\sin\theta + \cos\theta + 1)(\sin\theta - 1 + \cos\theta)\sec\theta\operatorname{cosec}\theta$

$= (\sin\theta + \cos\theta + 1)(\sin\theta + \cos\theta - 1)\ \sec\theta\operatorname{cosec}\theta$

$= \left[(\sin\theta + \cos\theta)^2 - 1^2\right]\sec\theta\operatorname{cosec}\theta \left[\because (a+b)(a-b) = \left(a^2 - b^2\right)\right]$

$= \left(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1\right)\sec\theta\operatorname{cosec}\theta \left[\because (a+b)^2 = a^2 + b^2 + 2ab\right]$

$= (1 + 2\sin\theta\cos\theta - 1)\sec\theta\operatorname{cosec}\theta \left[\because \sin^2 A + \cos^2 A = 1\right]$

$= 2\sin\theta\cos\theta \times \dfrac{1}{\cos\theta} \times \dfrac{1}{\sin\theta} = 2$

$=$ RHS

42. $\dfrac{\cot^2 A}{\operatorname{cosec}^2 A - 1} - 1$

$= \dfrac{\cot^2 A - \operatorname{cosec} A + 1}{\operatorname{cosec} A - 1}$

$= \dfrac{-\operatorname{cosec} A + \operatorname{cosec}\ ^2 A}{\operatorname{cosec} A - 1}$

$= \dfrac{\text{cosec } A(\text{cosec } A - 1)}{\text{cosec } A - 1}$

$= \text{cosec}A$

43. The L.H.S of the equation can be written as,

$\Rightarrow \dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\cos A}$

$\Rightarrow \dfrac{\cos^2 A - \sin^2 A}{\sin A \cos A}$

$\Rightarrow \dfrac{\cos^2 A - \left(1 - \cos^2 A\right)}{\sin A \cos A}$

$\Rightarrow \dfrac{\cos^2 A + \cos^2 A - 1}{\sin A \cos A}$

$\Rightarrow \dfrac{2\cos^2 A - 1}{\sin A \cos A}$

Since, L.H.S. = R.H.S. hence, proved that $\cot A - \tan A = \dfrac{2\cos^2 A - 1}{\sin A \cos A}$

44. Solving L.H.S of equation,

$\Rightarrow \dfrac{1}{\cos^2 A} + \dfrac{1}{\sin^2 A}$

$\Rightarrow \dfrac{\sin^2 A + \cos^2 A}{\cos^2 A \sin^2 A}$

By formula, $\sin^2 A + \cos^2 A = 1$

$\Rightarrow \dfrac{1}{\cos^2 A \sin^2 A}$

$\Rightarrow \dfrac{1}{\cos^2 A} \times \dfrac{1}{\sin^2 A}$

$\Rightarrow \sec^2 A \text{cosec}^2 A$

Since, L.H.S. = R.H.S.,

Hence, proved that $\sec^2 A + \text{cosec}^2 A = \sec^2 A \text{cosec}^2 A$.

45. The L.H.S. of the equation can be written as,

$\Rightarrow \sqrt{\left(\dfrac{1 - \sin A}{1 + \sin A}\right)}$

$\Rightarrow \sqrt{\dfrac{1 - \sin A}{1 + \sin A} \times \dfrac{1 - \sin A}{1 - \sin A}}$

$\Rightarrow \sqrt{\dfrac{(1 - \sin A)^2}{1 - \sin^2 A}}$

$\Rightarrow \sqrt{\dfrac{(1 - \sin A)^2}{\cos^2 A}}$

$\Rightarrow \dfrac{(1 - \sin A)}{\cos A}$

$\Rightarrow \dfrac{1}{\cos A} - \dfrac{\sin A}{\cos A}$

$\Rightarrow \sec A - \tan A$

Since, L.H.S. = R.H.S.

Hence, proved that $\sqrt{\left(\dfrac{1 - \sin A}{1 + \sin A}\right)} = \sec A - \tan A$.

46. Solving L.H.S. of the equation:

$\Rightarrow \left(1 + \tan^2 A\right) + \left(1 + \cot^2 A\right)$

$\Rightarrow \sec^2 A + \text{cosec}^2 A$

$\Rightarrow \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$

$\Rightarrow \frac{\sin^2 A + \cos^2 A}{\cos^2 A\sin^2 A}$

$\Rightarrow \frac{1}{\cos^2 A\sin^2 A}$

$\Rightarrow \frac{1}{\left(1-\sin^2 A\right)\sin^2 A}$

$\Rightarrow \frac{1}{\sin^2 A - \sin^4 A}$

Since, L.H.S. = R.H.S.

Hence, proved that $\left(1 + \tan^2 A\right) + \left(1 + \cot^2 A\right) = \left(\frac{1}{\sin^2 A - \sin^4 A}\right)$.

47. LHS = $\frac{\sec A - 1}{\sec A + 1} = \frac{\left(\frac{1}{\cos A}\right) - 1}{\left(\frac{1}{\cos A}\right) + 1}$ $\left[\because \sec A = \frac{1}{\cos A}\right]$

$= \frac{1-\cos A}{\cos A} \times \frac{\cos A}{1+\cos A}$

$= \frac{1-\cos A}{1+\cos A}$ = RH

Hence proved.

48. Total number of outcomes $= 5$ white +2 red +3 black balls $= 10$

Let A be the event of getting a red ball.

The number of favorable outcomes to the event $A = 2$

$\therefore P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{2}{10} = \frac{1}{5}$

Hence, the probability of drawing a red ball is $\frac{1}{5}$.

49. Number of red balls $= 5$

Let number of blue balls $= x$

$\therefore$ Total balls in the bag $= (5 + x)$

Probability of drawing a blue ball is double that of a red ball

Now probability of a blue ball $= \frac{x}{5+x}$

Probability of a red ball $= \frac{5}{5+x}$

According to the condition,

$\frac{x}{5+x} = \frac{2\times5}{5+x} \Rightarrow \frac{x}{5+x} = \frac{10}{5+x} \Rightarrow 5x + x^2 = 50 + 10x \Rightarrow x^2 + 5x - 10x - 50 = 0$

$\Rightarrow x^2 - 5x - 50 = 0 \Rightarrow x^2 - 10x + 5x - 50 = 0 \left\{ \begin{array}{r} \because -50 = -10\times5 \\ -5 = -10+5 \end{array} \right\}$

$\Rightarrow x(x-10) + 5(x-10) = 0 \Rightarrow (x-10)(x+5) = 0$

Either $x - 10 = 0$, then $x = 10$

or $x + 5 = 0$, then $x = -5$ which is not possible being negative

$\therefore$ Number of blue balls $= 10$

50. Vowels in 'TRIANGLE' = 'I', 'A', 'E'.

No. of letters in 'TRIANGLE' $= 8$.

Let $E_1$ be the event of choosing a vowel, then number of favourable outcomes to $E_1 = 3$

$P\left(E_1\right) = \frac{\text{No. of favourable outcomes to } E_1}{\text{Total no. of possible outcomes}} = \frac{3}{8}$.

Hence, the probability of choosing a vowel is $\frac{3}{8}$.

51. There are 365 days in a year, so Sarita ’s birthday can be at any day of 365 days in the year. Similarly, Hemlata’s birthday can be at any day of 365 days in the year. So, total number of possible outcomes =

365 × 365

i. If both have same birthday, then the number of outcomes favourable for their birthday = 365

$\therefore$ P (Sarita and Hemlata have same birthday)

$= \frac{365}{365 \times 365} = \frac{1}{365}$

ii. P (Sarita and Hemlata have different birthdays)

= 1 - P (Sarita and Hemlata have same birthday) [ $\because P(E) + P(\bar{E}) = 1$]

$= 1 - \frac{1}{365} = \frac{364}{365}$

**Section C**

52. Given, radius of a cone, r = 8 cm

and height of a cone, h = 6.912 cm

Now, volume of a cone

$\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times (8)^2 \times$ 6.912 cm

$= \frac{442.368}{3}\pi$ cm$^3$

Let $r_1$ be the radius of a sphere.

Then, according to the given condition,

Volume of cone = Volume of sphere

$\Rightarrow \frac{442.368}{3}\pi = \frac{4}{3}\pi r_1^3$

$\Rightarrow \frac{442.368}{4} = r_1^3 \Rightarrow r_1^3 = 110.592$

$\therefore$ $r_1$ = 4.8 cm

Hence, the radius of the sphere is 4.8 cm.

53. Internal radius of circular pipe $= \frac{\text{Diameter}}{2} = \frac{2}{2} = 1\ cm.$

Volume of water flowing through pipe per second = Area of cross section of pipe $x$ rate of flow of water

$= \pi r^2 \times 8\ m/s$

$= \pi r^2 \times 8 \times 100\ cm/s$

$= \frac{22}{7} \times (1)^2 \times 800$

$= \frac{22}{7} \times 800$

$= \frac{17600}{7}\ cm^3/s$

Volume of cylindrical tank $= \pi R^2 h$

$= \frac{22}{7} \times 40^2 \times h$

$= \frac{22}{7} \times 1600 \times h$

$= \frac{35200}{7} \times h$

Volume of water flowing through pipe in 30 minutes = Volume of cylindrical tank

(1 minute = 60 second so, 30 minutes = 30 × 60 = 1800 $s$)

$\Rightarrow 1800 \times \frac{17600}{7} = \frac{35200}{7} \times h$

$\Rightarrow \frac{31680000}{7} = \frac{35200}{7} \times h$

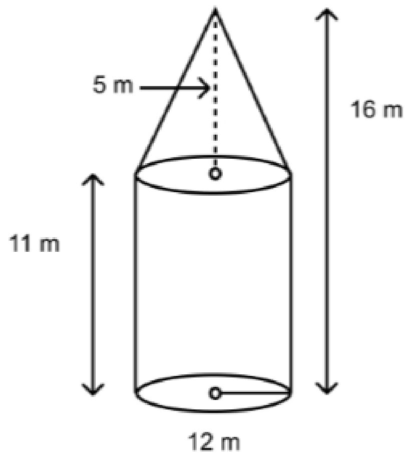
$\Rightarrow h = \frac{31680000}{7} \times \frac{7}{35200}$

$\Rightarrow h = 900\ cm.$

$\Rightarrow h = 9\ m.$

Hence, the increase in the water level in 30 minutes is 9 m.

54. The figure of the circus tent is shown below:




Diameter of cylindrical portion = 24 m

Radius of cylindrical portion $(r) = \frac{\text{Diameter}}{2}$

$= \frac{24}{2} = 12\ m.$

Height of the cylindrical part, $H = 11\ m.$

Since vertex of cone is 16 m above the ground, height of cone, h $= 16 - 11 = 5\ m.$

$h = 5\ m.$

Radius of cone $= 12\ m.$

$\therefore$ Radius of cone is also equal to $r.$

Slant height of the cone, $l = \sqrt{h^2 + r^2}.$

$l = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13\ m.$

Area of canvas used to make the tent = Curved surface area of the cylindrical part + Curved surface area of the cone.

Area of the canvas used to make the tent $= 2\pi rH + \pi rl = \pi r(2H + \text{l}).$

Putting values we get,

Area of the canvas $= \frac{22}{7} \times 12 \times (2 \times 11 + 13)$

$= \frac{22}{7} \times 12 \times 35$

$= 22 \times 12 \times 5$

$= 1320\ m^2$

Hence, the area of the canvas used to make the tent is 1320 $m^2$.

55. Dimension of cylinder:

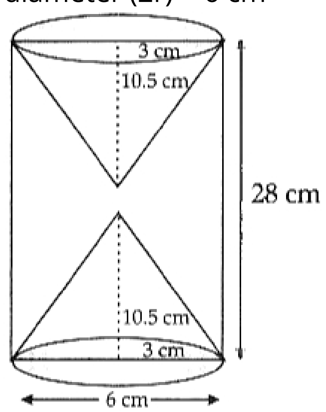
height (h) = 28 cm

diameter (2r) = 6 cm

Dimension of conical cavities:

height (H) = 10.5 cm

diameter (2r) = 6 cm



Volume of remaining solid = volume of cylinder - 2 $\times$ volume of cavities

$= \pi r^2 h - 2 \times \frac{1}{3}\pi r^2 H$

$= \pi r^2\left(h - \frac{2}{3}\mathrm{H}\right)$

$= \frac{22}{7} \times 3 \times 3\left(28 - \frac{2}{3} \times 10.5\right)$

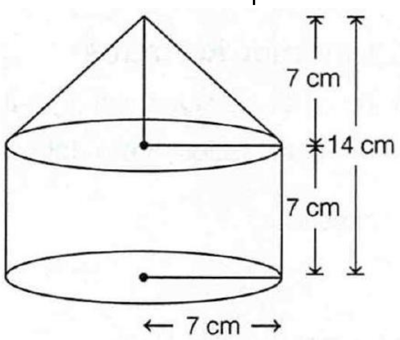
$= \frac{22}{7} \times 3 \times 3(28 - 7)$

$= \frac{22}{7} \times 3 \times 3 \times 21$

$= 22 \times 3 \times 3 \times 3$

Volume of remaining solid = 594 $cm^3$

56. Given, radius of cylindrical part
= Radius of conical part = $r = 7\ m$




Height of cylindrical part $(H) = 7\ m$
and height of conical part $(h) = 14 - 7 = 7\ m$

i. Now, quantity of air inside the tent
= Volume of cylindrical part + Volume of conical part

$= \pi r^2 H + \frac{1}{3}\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 7 + \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= 22 \times 49 + \frac{1}{3} \times 22 \times 49$

$= 1078 + 359.33 = 1437.33\ m^3$

Hence, the quantity of air contained inside the tent is $1437.33\ m^3$.

ii. Let the radius of required sphere be $R$.
According to the question,
Volume of sphere = Quantity of air inside the tent

$\therefore \frac{4}{3}\pi R^3 = \pi r^2 H + \frac{1}{3}\pi r^2 h$

$\Rightarrow \frac{4}{3}R^3 = r^2 H + \frac{1}{3}r^2 h$

$\Rightarrow \frac{4}{3}R^3 = \frac{3r^2 H + r^2 h}{3}$

$\Rightarrow 4R^3 = 3r^2 H + r^2 h$

$\Rightarrow R^3 = \frac{3 \times 7 \times 7 \times 7 + 7 \times 7 \times 7}{4}$

$\Rightarrow R^3 = \frac{1372}{4} \Rightarrow R^3 = 343$

$\Rightarrow R^3 = 7^3 \Rightarrow R = 7\ m$

Hence, the radius of sphere is 7 m.

57. $\because$ Common radius (r) = 7 cm
and height of cylinder (h) = height of cone (h) = 4 cm
Volume of the solid = Volume of hemisphere + volume of cylinder + volume of cone

Volume of the solid = $\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{1}{3}\pi r^2 h$

= $\frac{\pi r^2}{3}$[2r + 3h + h]

= $\frac{28 \times 7^2}{7 \times 3}$[2 $\times$ 7 + 4 $\times$ 4] [ $\because$ Given, r = 7 cm and h = 4 cm]

= $\frac{22 \times 7}{3}$[14 + 16]

= $\frac{22 \times 7}{3}$ $\times$ 30 = 1540 cm$^3$.

58. Volume of the whole cone of metal A

$= \frac{1}{3}\pi r^2\ h$

$= \frac{1}{3} \times \pi \times 6^2 \times 10$

$= 120\pi$

Volume of the cone with metal B

$= \frac{1}{3}\pi r^2\ h$

$= \frac{1}{3} \times \pi \times 3^2 \times 4$

$= 12\pi$

Final Volume of cone with metal $A = 120\pi - 12\pi = 108\pi$

Volume of cone with metal $A$

Volume of cone with metal $B$

$= \frac{108\pi}{12\pi} = \frac{9}{1}$

59. $LHS = (r\cos A\cos B)^2 + (r\cos A\sin B)^2 + (r\sin A)^2$

$= r^2\cos^2 A\cos^2 B + r^2\cos^2 A\sin^2 B + r^2\sin^2 A$

$= r^2\cos^2 A\left(\cos^2 B + \sin^2 B\right) + r^2\sin^2 A$

$= r^2\left(\cos^2 A + \sin^2 A\right) = r^2 = RHS$

60. Given, 1 + sin$^2$ $\theta$ = 3 sin $\theta$ cos $\theta$

On dividing both sides by cos$^2$ $\theta$, we get

$\frac{1}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = 3\frac{\sin\theta\cos\theta}{\cos^2\theta}$

$\Rightarrow$ sec$^3$ $\theta$ + tan$^2$ $\theta$ = 3tan $\theta$

$\Rightarrow$ (1 + tan$^2$ $\theta$) + tan$^2$ $\theta$ = 3tan $\theta$ [ $\because$ sec$^2$ A = 1 + tan$^2$ A]

$\Rightarrow$ 2 tan$^2$ $\theta$ - 3 tan $\theta$ + 1 = 0

$\Rightarrow$ 2 tan$^2$ $\theta$ - 2 tan $\theta$ - tan $\theta$ + 1 = 0 [by factorisation method]

$\Rightarrow$ 2 tan $\theta$ (tan $\theta$ - 1) - 1(tan $\theta$ - 1) = 0

$\Rightarrow$ (tan $\theta$ - 1) (2tan $\theta$ - 1) = 0

$\Rightarrow$ tan $\theta$ - 1 = 0 or 2 tan $\theta$ - 1 = 0

$\Rightarrow$ tan $\theta$ = 1 or tan $\theta$ = $\frac{1}{2}$

61. tan$\theta$ = $\frac{20}{21}$, then

prove that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

$\because \tan\theta = \frac{20}{21} = \frac{p}{b}$

p = 20, b = 21

$h = \sqrt{20^2 + 21^2}$

h = 29

$\sin\theta = \frac{20}{29}$ and $\cos\theta = \frac{21}{29}$

LHS: $\frac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta}$

$\Rightarrow \frac{1 - \frac{20}{29} + \frac{21}{29}}{1 + \frac{20}{29} + \frac{21}{29}}$

$\Rightarrow \frac{\frac{29 - 20 + 21}{29}}{\frac{29 + 20 + 21}{29}} = \frac{30}{70} = \frac{3}{7}$ = RHS

Hence proved.

62. i. LHS = (sec A - sin A) (cosec A + cos A)

$= \left(\frac{1}{\cos A} - \sin A\right)\left(\frac{1}{\sin A} + \cos A\right)$

$= \left(\frac{1 - \sin A \cos A}{\cos A}\right)\left(\frac{1 + \sin A \cos A}{\sin A}\right)$

$= \frac{\left(1 - \sin^2 A \cos^2 A\right)}{\sin A \cos A}$ [ $\because$ (a - b)(a + b) = $a^2 - b^2$]

$= \frac{\sin^2 A + \cos^2 A - \sin^2 A \cos^2 A}{\sin A \cos A}$ [ $\because$ $\sin^2\theta + \cos^2\theta = 1$]

$= \frac{\sin^2 A - \sin^2 A \cos^2 A + \cos^2 A}{\sin A \cos A}$

$= \frac{\sin^2 A\left(1 - \cos^2 A\right) + \cos^2 A}{\sin A \cos A}$

$= \frac{\sin^2 A \sin^2 A + \cos^2 A}{\sin A \cos A} = \frac{\sin^4 A + \cos^2 A}{\sin A \cos A}$

$= \frac{\sin^4 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A} = \frac{\sin^3 A}{\cos A} + \frac{\cos A}{\sin A}$

$= \sin^2 A \cdot \frac{\sin A}{\cos A}$ + cot A

= $\sin^2$ A $\cdot$ tan A + cot A = RHS

Hence proved.

ii. LHS = (1 + cot A + tan A) (sin A - cos A)

$= \left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)$ (sin A - cos A)

$= \left(\frac{\sin A \cos A + \cos^2 A + \sin^2 A}{\sin A \cos A}\right)$ (sin A - cos A)

$= \frac{(\sin A - \cos A)\left(\sin^2 A + \cos^2 A + \sin A \cos A\right)}{\sin A \cos A}$

$= \frac{\sin^3 A - \cos^3 A}{\sin A \cos A}$ [ $\because$ (a - b)($a^2 + b^2$ + ab) = $a^3 - b^3$] ...(i)

and RHS = $\frac{\sec A}{\csc^2 A} - \frac{\csc A}{\sec^2 A} = \frac{\frac{1}{\cos A}}{\frac{1}{\sin^2 A}} - \frac{\frac{1}{\sin A}}{\frac{1}{\cos^2 A}}$

$= \frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A} = \frac{\sin^3 A - \cos^3 A}{\cos A \sin A}$ ...(ii)

From Eqs. (i) and (ii),

LHS = RHS

Hence proved.

63. L.H.S.

$= \dfrac{(1+\sin\theta)^2+(1-\sin\theta)^2}{2\cos^2\theta}$

$= \dfrac{1^2+\sin^2\theta+2\sin\theta+1^2+\sin^2\theta-2\sin\theta}{2\cos^2\theta}$

$= \dfrac{2+2\sin^2\theta}{2\cos^2\theta} = \dfrac{1+\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \sec^2\theta + \tan^2\theta =$ R.H.S. Hence Proved.

64. R.H.S. $= \dfrac{1-\cos A}{1+\cos A}$

$= \dfrac{1-\cos A}{1+\cos A} \times \dfrac{1-\cos A}{1-\cos A}$

$= \dfrac{(1-\cos A)^2}{1-\cos^2 A}$

$= \dfrac{(1-\cos A)^2}{\sin^2 A}$

$= \left(\dfrac{1-\cos A}{\sin A}\right)^2$

$= (\text{cosec}A - \cot A)^2$

$= (\cot A - \text{cosec}A)^2$

$=$ L.H.S

65. (1 + cot $\theta$ - cosec $\theta$) (1 + tan $\theta$ + sec $\theta$) = 2

L.H.S.

= (1 + cot $\theta$ - cosec $\theta$) (1 + tan $\theta$ + sec $\theta$)

$= \left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)\left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)$

$= \left(\dfrac{\sin\theta+\cos\theta-1}{\sin\theta}\right)\left(\dfrac{\cos\theta+\sin\theta+1}{\cos\theta}\right)$

$= \dfrac{1}{\sin\theta\cos\theta}\left[(\sin\theta + \cos\theta)^2 - 1^2\right]$

$= \dfrac{1}{\sin\theta\cos\theta}$[sin$^2\theta$ + cos$^2\theta$ + 2sin $\theta$ cos $\theta$ - 1]

$= \dfrac{1}{\sin\theta\cos\theta}$ (1 + 2 sin $\theta$ cos $\theta$ - 1)

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

= 2

66. Number of possible outcomes = number of days in the month $= 31$

$n(S) = 31$

i. E = event of selection of a Wednesday $= [1, 8, 15, 22, 29]$

$n(E) - 5$

Probability of selection of a Wednesday $-P(S) = \dfrac{n(E)}{n(S)} = \dfrac{5}{31}$

ii. E = event of selection of a Friday $= [3, 10, 17, 24, 31]$

$n(E) = 5$

Probability of selection of a Friday $-P(S) = \dfrac{n(E)}{n(S)} = \dfrac{5}{31}$

iii. E = event of selection of a Tuesday or a Saturday = $\{4, 7, 11, 14, 18, 21, 25, 28\}$

n(E) = 8

Probability of selection of a Tuesday or a Saturday $= P(S) = \dfrac{n(E)}{n(S)} = \dfrac{8}{31}$

67. A number is selected at random it means that all outcomes are equally likely.

Sample space $= \{1, 2, 3, \ldots, 35\}$, which has 35 equally likely outcomes.

i. Let A be the event 'the number is prime', then
$A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31\}$.
∴ The number of favourable outcomes to the event $A = 11$.
$\therefore P(\text{ prime number }) = \frac{11}{35}$.
Hence, the probability that the number selected is a prime number is $\frac{11}{35}$.

ii. Let B be the event 'the number is multiple of 7', then
$B = \{7, 14, 21, 28, 35\}$.
∴ The number of favourable outcomes to the event $B = 5$.
$\therefore P(\text{ multiple of } 7) = \frac{5}{35} = \frac{1}{7}$.
Hence, the probability that the number selected is a multiple of 7 is $\frac{1}{7}$.

iii. Let C be the event 'the number is multiple of 3 or 5', then
C = {3, 5, 6, 9, 10, 12, 15, 18, 20, 21, 24, 25, 27, 30, 33, 35}.
∴ The number of favourable outcomes to the event $C = 16$.
$\therefore P(\text{ multiple of 3 or 5}) = \frac{16}{35}$.
Hence, the probability that the number selected is a multiple of 3 or 5 is $\frac{16}{35}$.

68. Total no. of Sample space = 62 = 36
Let 'E' be the event of getting a sum more than 9.
E = {getting a sum of 10 + getting a sum 11 + getting a sum}
E = {(4, 6), (5, 5), (6, 4) or (5, 6), (6, 5) or (6, 6)}
n(E) = 6
n(S) = 36
P(E) = $\frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

69. Let E and F denote the events that Niharika and Shreya win the match, respectively. It is clear that, if Niharika wins the match, then Shreya losses the match and if Shreya wins the match, then Niharika losses the match. Thus, E and F are complementary events.
∴ P(E) + P(F) = 1
Since, probability of Niharika 's winning the match, i.e. P(E) = 0.62
∴ Probability of Shreya's winning the match,
P(F) = P (Niharika losses the match)
= 1 - P(E) [ $\because P(E) + P(F) = 1$]
= 1 - 0.62 = 038

**Section D**

70. Given, radius of cylinder = radius of cone = r = 3 cm
height of the cylinder = h = 12 cm
Slant height of the cone = l = 5 cm
$h_1$ = height of cone = $\sqrt{l^2 - r^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
Total surface area of rocket = curved surface area of cylinder + curved surface area of cone + Area of base of cylinder
= $2\pi rh + \pi rl + \pi r^2 = \pi r(2h + l + r)$
= 3.14 × 3 (2 × 12 + 5 + 3) = 301.44 $cm^2$
Volume of rocket = volume of cylinder + volume of cone
$= \pi r^2 h + \frac{1}{3}\pi r^2 h_1$
$= \pi r^2\left(h + \frac{1}{3} \times h_1\right) = 3.14 \times 3^2 \times \left(12 + \frac{1}{3} \times 4\right)$
$= 3.14 \times 9 \times \frac{40}{3}$ = 376.8 $cm^3$

71. Given, radius of hemisphere $r = 7\ cm$.

The volume of hemisphere, $V_1 = \frac{2}{3}\pi r^3$

$\Rightarrow V_1 = \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \left[\because \pi = \frac{22}{7}\right]$

$\Rightarrow V_1 = 718.66\ cm^3$

Now, the volume of the cone

$V_2 = \frac{1}{3}\pi r^2 h \Rightarrow V_2 = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 4$

[ $\because$ given, cone's height $(h) = 4\ cm$]

$\Rightarrow V_2 = \frac{22 \times 28}{3} = 205.33\ cm^3$

$\therefore$ Total volume of solid, $V = V_1 + V_2$

$= 718.66 + 205.33$

$= 923.99\ cm^3$

We know that the volume of water displaced is equal to the volume of the solid.

Let $h\ cm$ rises water level.

The volume of a cylinder is $V = \pi r^2 h$

On putting $r = 14$ (given) and $V = 923.99$, we get

$923.99 = \frac{22}{7} \times 14 \times 14 \times h$

$\Rightarrow h = 1.49\ cm \approx 1.5\ cm$

Hence, the rise in the water level is 1.5 cm.

72. For cylinder:

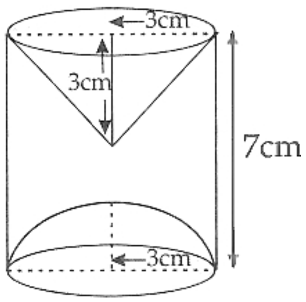
Height of cylinder, (H) = 7 cm

and radius of cylinder (r) = 3 cm

$\therefore$ Total volume of cylinder = $\pi r^2 H$

$= \frac{22}{7} \times 3 \times 3 \times 7$

$= 198\ cm^3$




In case of cone:

Radius of a cone (r) = 3 cm

and height of a cone (h) = 3 cm

$\therefore$ Volume of a cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3$

$= \frac{198}{7}\ cm^3$

In case of hemisphere:

Radius of hemisphere (r) = 3 cm

$\therefore$ Volume of hemisphere = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$

$= \frac{396}{7}\ cm^3$

Total volume of cone and hemisphere

$= \left(\frac{198}{7} + \frac{396}{7}\right)\text{cm}^3$

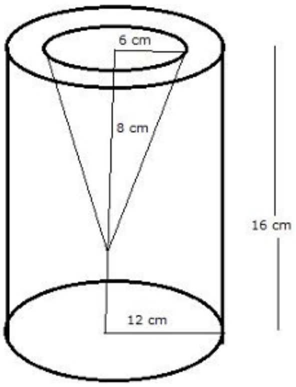
$= \frac{594}{7}$ cm$^3$

Now, the volume of the remaining solid = Total volume of a cylinder - Total volume of cone and hemisphere

$= \left(198 - \frac{594}{7}\right)\text{cm}^3$

$= \frac{792}{7}$ = 113 cm$^3$ (Approx.)

73.




Radius of solid cylinder $(R) = 12\ cm$

and Height $(H) = 16\ cm$

$\therefore$ Volume $= \pi R^2 H$

$= \frac{22}{7} \times 12 \times 12 \times 16$

$= \frac{50688}{7}\ cm^3$

Radius of cone $(r) = 6\ cm$, and height $(h) = 8\ cm$.

$\therefore$ Volume $= \frac{1}{3}\pi r^2\ h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

$= \frac{2112}{7}\ cm^3$

i. Volume of remaining solid

$= \frac{50688}{7} - \frac{2112}{7}$

$= \frac{48567}{7}$

$= 6939.43^2\ cm^3$

ii. Slant height of cone $\ell = \sqrt{h^2 + r^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10\ cm$

Therefore, total surface area of remaining solid = curved surface area of cylinder + curved surface area of cone + base area of cylinder + area of circular ring on upper side of cylinder

$= 2\pi Rh + \pi r\ell + \pi R^2 + \pi\left(R^2 - r^2\right)$

$= \left(2 \times \frac{22}{7} \times 12 \times 16\right) + \left(\frac{22}{7} \times 6 \times 10\right) + \left(\frac{22}{7} \times 12 \times 12\right) + \left(\frac{22}{7}\left(12^2 - 6^2\right)\right)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{22}{7}(144 - 36)$

$= \frac{8448}{7} + \frac{1320}{7} + \frac{3168}{7} + \frac{2376}{7}$

$= \frac{15312}{7}$

$= 2187.43\ cm^3$

74. $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\mathrm{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta}{\left(1-\cos^4\theta\right)} + \frac{\sin^2\theta}{\left(1-\sin^4\theta\right)}\right)\sin^2\theta\cos^2\theta$

$\because$ We know that

$a^2 - b^2 = (a - b)(a + b)$

$= \left(\frac{\cos^2\theta}{\left(1-\cos^2\theta\right)\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\left(1-\sin^2\theta\right)(1+\sin\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta}{\sin^2\theta\left(1+\cos^2\theta\right)} + \frac{\sin^2\theta}{\cos^2\theta\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$

$\because$ $1 - \cos^2\theta = \sin^2\theta$

$1 - \cos^2\theta = \cos^2\theta$

$= \left(\frac{\cos^4\theta\left(1+\sin^2\theta\right) + \sin^2\theta\left(1+\cos^2\theta\right)}{\sin^2\theta\cos^2\theta\left(1+\cos^2\theta\right)\left(1+\sin^2\theta\right)}\right)\sin^2\theta\cos^2\theta$

$= \frac{\cos^4\theta + \cos^4\theta\sin^2\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{1+\cos^2\theta+\sin^2\theta+\cos^2\theta\sin^2\theta}$

$= \frac{\left(\cos^4\theta+\sin^4\theta\right) + \left(\cos^4\theta\sin^2\theta+\sin^4\theta\cos^2\theta\right)}{1+1+\cos^2\theta\sin^2\theta}$ ( $\because$ $\sin^2\theta + \cos^2\theta = 1$) and

$a^2 + b^2 = (a + b)^2 - 2ab$

$= \frac{\left(\cos^2\theta+\sin^2\theta\right)^2 - 2\cos^2\theta\sin^2\theta + \cos^2\theta\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}{2+\cos^2\theta\sin^2\theta}$

$= \frac{1-2\sin^2\theta\cos^2\theta+\cos^2\theta\sin^2\theta}{2+\cos^2\theta\sin^2\theta}$ { $\because$ $\cos^2\theta + \sin^2\theta = 1$}

$= \frac{1-\sin^2\theta\cos^2\theta}{2+\cos^2\theta\sin^2\theta}$

= R.H.S. Proved

75. $\sqrt{3}\cot^2\theta - 4\cot\theta + \sqrt{3} = 0$

Let $\cot\theta = x$

$\Rightarrow \sqrt{3}x^2 - 4x + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}x^2 - 3x - x + \sqrt{3} = 0$

$\Rightarrow \sqrt{3}x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$

$\Rightarrow (\sqrt{3}x - 1)(x - \sqrt{3}) = 0$

x = $\frac{1}{\sqrt{3}}$ or $\sqrt{3}$

i.e. cot $\theta$ = $\sqrt{3}$ or cot $\theta$ = $\frac{1}{\sqrt{3}}$

$\theta = 30^\circ$ or $\theta = 60^\circ$

if $\theta = 30^\circ$, then

$\cot^2 30^\circ + \tan^2 30^\circ = (\sqrt{3})^2 + (\frac{1}{\sqrt{3}})^2$

$= 3 + \frac{1}{3}$

$= \frac{10}{3}$

if $\theta = 60^\circ$, then

$\cot^2 60^\circ + \tan^2 60^\circ = (\sqrt{3})^2 + (\frac{1}{\sqrt{3}})^2$

$= 3 + \frac{1}{3}$

$= \frac{10}{3}$

Hence, value of $\cot^2\theta + \tan^2\theta = \frac{10}{3}$

## New Stewart School, Cuttack

## QUESTIONS BANK CLASS 10
### Class 10 - Mathematics

**Section A**

1. If $3 : x :: 6 : 18$ , then the value of $x$ is **[1]**

a) 9 b) 18

c) 28 d) 20

2. If $a, b, c$ and $d$ are proportional, then $\frac{a+b}{a-b}$ is equal to **[1]**

a) $\frac{c-d}{c+d}$ b) $\frac{c}{d}$

c) $\frac{d}{c}$ d) $\frac{c+d}{c-d}$

3. If $x, 5.4, 5, 9$ are in proportion, then $x$ is **[1]**

a) 25 b) $\frac{25}{3}$

c) 3 d) 9.72

4. The mean proportion between 0.02 and 0.32 is: **[1]**

a) 0.08 b) 0.16

c) 0.34 d) 0.3

5. The mean proportion between x and y is 6. The third proportional to x and y is 48. Then x : y equals: **[1]**

a) 1 : 9 b) 1 : 4

c) 1 : 6 d) 1 : 3

6. The 8th term from the end of the A.P. 7, 10, 13, ..., 184 is: **[1]**

a) 163 b) 157

c) 160 d) 166

7. The $4^{\text{th}}$ term of a G.P. is 16 and the $7^{\text{th}}$ term is 128, then its common ratio is equal to: **[1]**

a) -1 b) -2

c) 2 d) 1

8. The sum of 25 terms of the A.P., $-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \ldots$ is **[1]**

a) 0 b) -50

c) $-\frac{50}{3}$ d) $-\frac{2}{3}$

9. If $\mathrm{k}, 2(\mathrm{k}+1), 3(\mathrm{k}+1)$ are three consecutive terms of a G.P., then the value of $k$ is **[1]**

a) -1 b) 1

c) -4 d) 4

10. Point (5, 6) is reflected in a line to get the point (-5, 6). The line of reflection is: **[1]**

a) y = 0
b) x = 5
c) x-axis
d) y-axis

11. The reflection of the point P(-2, 3) in the x-axis is: **[1]**

a) (-2, 3)
b) (2, 3)
c) (-2, -3)
d) (2, -3)

12. The reflection of the point A(4, -7) in the y-axis is: **[1]**

a) (-4, -7)
b) (-4, 7)
c) (4, 7)
d) (4, -7)

13. The distance of the point P(-3, -4) from the x-axis is: **[1]**

a) 5 units
b) 6 units
c) 3 units
d) 4 units

14. A point lies on the x-axis at a distance of 5 units from the y-axis to the left side of the origin. The co-ordinates of this point are: **[1]**

a) (0, -5)
b) (0, 5)
c) (5, 0)
d) (-5, 0)

15. The mid-point of the line segment joining the points $(-3,2)$ and $(7,6)$ is: **[1]**

a) $(2,4)$
b) $(-2,-4)$
c) $(-2,4)$
d) $(4,2)$

16. If the point $P(6,2)$ divides the line segment joining $A(6,5)$ and $B(4,y)$ in the ratio $3:1$, then the value of $y$ is: **[1]**

a) 1
b) 4
c) 3
d) 2

17. In what ratio is the line segment joining the points $\mathrm{P}(-4,2)$ and $Q(8,3)$ divided by $y$-axis? **[1]**

a) $2:1$
b) $3:1$
c) $1:2$
d) $1:3$

18. Point $P\left(\frac{a}{8},4\right)$ is the mid-point of the line segment joining the points $\mathrm{A}(-5,2)$ and $\mathrm{B}(4,6)$. Then, the value of a is: **[1]**

a) 4
b) -8
c) 2
d) -4

19. The slope of a line perpendicular to the line $3x=4y+11$ is **[1]**

a) $\frac{4}{3}$
b) $-\frac{3}{4}$
c) $-\frac{4}{3}$
d) $\frac{3}{4}$

20. Equation of a line passing through the intersection of the lines $x-y=3$ and $x+y=0$ with inclination $45^{\circ}$ is: **[1]**

a) $x+y=3$ b) $x-y=3$

c) $y=3x+1$ d) $y-x=3$

21. The equation of a line with $x$-intercept 7 and $y$-intercept -7 is : **[1]**

a) $x+y+7=0$ b) $x-y=7$

c) $x+y=7$ d) $y-x=7$

22. The line passing through point $(1,1)$ and making $y$-intercept equal to 3 has equation: **[1]**

a) $2x-y+3=0$ b) $2x-y=3$

c) $2y+x=3$ d) $2x+y=3$

**Section B**

23. **Assertion (A):** If $3:x::2:18$, then the value of $x$ is 27. **[1]**

**Reason (R):** If four quantities $a, b, c$ and $d$ form a proportion, then we get $\frac{a}{b}=\frac{c}{d}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

24. **Assertion (A):** The sum of first 99 natural numbers is 4950. **[1]**

**Reason (R):** The sum of first $n$ natural numbers is $\frac{n(n+1)}{2}$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

25. **Assertion (A):** The coordinates of the centroid of a triangle, whose vertices are $A(-1,3), B(1,-1)$ and $C(5,1)$, is $\left(\frac{5}{3},1\right)$. **[1]**

**Reason (R):** If the vertices of a $\triangle ABC$ are $A\left(x_1,y_1\right), B\left(x_2,y_2\right)$ and $C\left(x_3,y_3\right)$, then centroid of a triangle is $\left(\frac{x_1+x_2+x_3}{2}, \frac{y_1+y_2+y_3}{2}\right)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

26. **Assertion (A):** The slope of the line perpendicular to the line passing through the points (2, 5) and (-3, 6) is given by 5. **[1]**

**Reason (R):** The product of the slopes of two perpendicular lines is always -1.

a) Both A and R are true

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

**Section C**

27. **Statement I:** If $a, 4, 12$ and $b$ are in continued proportion, then the values of $a$ and $b$ are $\frac{4}{3}$ and 36. **[1]**

**Statement II:** If $\frac{a^2+b^2}{a^2-b^2}=\frac{17}{8}$, then the value of $a:b$ is $8:3$.

a) Both the Statements are false.

b) Statement I is false and Statement II is true.

c) Both the Statements are true.

d) Statement I is true and Statement II is false.

28. **Statement I:** If $p$ th, $q$ th and $r$ th terms of a GP are in GP, then $p, q, r$ are in AP. **[1]**
**Statement II:** In GP a, ar, ar $^2, \ldots$, if $r=1$, then $S_n=a+a+a+\ldots+n$ terms $=na$.

a) Statement I is false and Statement II is true.

b) Statement I is true and Statement II is false.

c) Both the Statements are false.

d) Both the Statements are true.

29. **Statement I:** If $P(x, y)$ is a point in the coordinate plane, then the point $P^{\prime}(y, x)$ is the reflection of point $P$ in the origin. **[1]**
**Statement II:** The reflection of a point $P(x, y)$ in the line $x=a$ is $P^{\prime}(x,-y+2a)$.
Which of the following is valid?

a) Both the Statements are true.

b) Statement I is false and Statement II is true.

c) Statement I is true and Statement II is false.

d) Both the Statements are false.

30. **Statement I:** If the vertices of a triangle are (6, 4), $(3,2)$ and $(1,6)$, then the coordinates of the point of intersection of the median of the triangle are $\left(\frac{10}{3}, 4\right)$. **[1]**
**Statement II:** If the three vertices of a triangle are $(6,11),(8,3)$ and $(x, 4)$ and its centroid is $(9,6)$, then value of $x$ is 13.

a) Statement I is true and Statement II is false.

b) Both the Statements are true.

c) Both the Statements are false.

d) Statement I is false and Statement II is true.

31. **Statement I:** The inclination of the line passing through the points $A(-2,3)$ and $B(2,7)$ is $45^{\circ}$. **[1]**
**Statement II:** The slope of the line parallel to $AB$ when the line passing through the points $A(2,3)$ and $B(-5,0)$ is $\frac{7}{3}$.

a) Statement I is true and Statement II is false.

b) Statement I is false and Statement II is true.

c) Both the Statements are false.

d) Both the Statements are true.

## Section D

32. If $a, b, c$ are in continued proportion, prove that $(a+b+c)(a-b+c)=a^2+b^2+c^2$. **[3]**

33. If $a, b, c, d$ are in proportion, prove that $\left(a^4+c^4\right):\left(b^4+d^4\right)=a^2 c^2: b^2 d^2$. **[3]**

34. If (4a + 5b)(4c - 5d) = (4a - 5b)(4c + 5d), prove that a, b, c, d are in proportion. **[3]**

35. Using properties of proportion, find x from the following equations $\frac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}+\sqrt{x-10}}=\frac{5}{2}$. **[3]**

36. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ... 185. **[3]**

37. If 1701 is the $n$th term of the geometric progression GP 7, 21, 63, ..., find **[3]**

i. the value of $n$

ii. hence find the sum of the $n$ terms of the GP.

38. If the numbers n - 2, 4n - 1 and 5n + 2 are in A.P., find the value of n. **[3]**

39. Find the 8th term of a G.P., if its common ratio is 2 and 10th term is 768. **[3]**

40. Use a graph paper for this question. Take $2\ cm = 1$ unit along both the axes. **[3]**

i. Plot the points $A(0,4), B(2,2), C(5,2), D(4,0)$ and $E(0,0)$ is the origin.
ii. Reflect $B, C, D$ on the $Y$-axis and name them as $B', C'$ and $D'$, respectively.
iii. Join the points $A, B, C, D, D', C', B'$ and $A$ in order and give a geometrical name to the closed figure.

41. i. Point $P(2,-3)$ on reflection becomes $P'(2,3)$. Name the line of reflection (say $L_1$). **[3]**
ii. Point $P'$ is reflected to $P''$ along the line $(L_2)$, which is perpendicular to the line $L_1$ and passes through the point, which is invariant along both axes. Write the coordinates of $P''$.
iii. Name and write the coordinates of the point of intersection of the lines $L_1$ and $L_2$.
iv. Point $P$ is reflected to $P''$ on reflection through the point named in the answer of part I of this question. Write the coordinates of $P'''$. Comment on the location of the points $P''$ and $P'''$.

42. Find the mid-point of the line segment joining the points: **[3]**

i. (-6, 7) and (3, 5)
ii. (5, -3) and (-1, 7)

43. Determine the ratio in which the line $y = 2 + 3x$ divides the line segment $AB$ joining the points $A(-3,9)$ and $B(4,2)$. **[3]**

44. Points A and B have co-ordinates (3, 5) and (x, y) respectively. The mid-point of AB is (2, 3). Find the values of x and y. **[3]**

45. Find the mid-point of the line segment joining the points (3, -1) and (7, 9). **[3]**

46. Find a point P which divides internally the line segment joining the points A(-3, 9) and B(1, -3) in the ratio 1 : 3. **[3]**

47. Find the equation of a line parallel to the x-axis and passing through the point (–3, 2). **[3]**

48. Find the gradient of a line passing through the following pairs of points : **[3]**

i. (0, -2), (3, 4)
ii. (3, -7), (-1, 8).

49. Find the equation of a line parallel to the y-axis and passing through the point (-7, 5). **[3]**

50. Find the equation of a line whose: **[3]**

i. Slope $= \frac{3}{4}$ and y-intercept $= -4$
ii. Gradient $= \sqrt{3}$ and y-intercept $= \frac{-2}{3}$

51. Find '$a$' if $A(2a+2,3), B(7,4)$ and C$(2a+5,2)$ are collinear. **[3]**

52. If $a, b, c, d$ are in continued proportion, prove that $(a+d)(b+c)-(a+c)\ (b+d)=(b-c)^2$ **[4]**

53. If $(3x-9):(5x+4)$ is the triplicate ratio of $3:4$, find the value of $x$. **[4]**

54. If $q$ is the mean proportional between $p$ and $r$, show that: **[4]**
$pqr(p+q+r)^3 = (pq+qr+rp)^3$.

55. If x + 5 is the mean proportional between x + 2 and x + 9; find the value of x. **[4]**

56. The sum of the first three terms of an Arithmetic Progression (A.P.) is 42 and the product of the first and third term is 52. Find the first term and the common difference. **[4]**

57. The 4th term of a A.P. is 22 and 15th term is 66. Find the first term and the common difference. **[4]**

Hence, find the sum of the series to 8 terms.

58. The 4th term of a G.P is 16 and the 7th term is 128. Find the first term and common ratio of the series. **[4]**

59. The 2nd and 45th term of an arithmetic progression are 10 and 96 respectively. Find the first term and the common difference and hence find the sum of the first 15 terms. **[4]**

60. The points A(4, -11), B(5, 3), C(2, 15) and D(1, 1) are the vertices of a parallelogram. If the parallelogram is reflected in the y-axis and then in the origin, find the coordinates of the final images. Check whether it remains a parallelogram. Write down a single transformation that brings the above change. **[4]**

61. Points A and B have coordinates (1, -3) and (2, 4). Find the following. **[4]**
   i. The image A' of A under the reflection in the line x = 0.
   ii. The image B' of B under the reflection of the line AA'.

62. Find the co-ordinates of the centroid of $\triangle PQR$ whose vertices are $P(6,3), Q(-2,5)$ and $R(-1,7)$. **[4]**

63. The mid-point of the line-segment joining (4a, 2b – 3) and (-4, 3b) is (2, -2a). Find the values of a and b. **[4]**

64. P (1, -2) is a point on the line segment A (3, - 6) and B (x, y) such that AP : PB is equal to 2 : 3. Find the coordinates of B. **[4]**

65. Find the coordinates of the points of trisection of the line segment joining the points (5, -3) and (2, -9). **[4]**

66. In what ratio, does the point (5, 4) divide the line segment joining the points (2, 1) and (7, 6)? **[4]**

67. Two vertices of a triangle are (3, - 5) and (- 7, 4). If its centroid is (2, -1), then find the third vertex. **[4]**

68. $ABC$ is a triangle whose vertices are $A(1,-1), B(0,4)$ and $C(-6,4)$. $D$ is the mid-point of $BC$. Find the **[4]**
   i. coordinates of $D$.
   ii. equation of the median $AD$.

69. A (7, -1), B (4, 1) and C (-3, 4) are the vertices of a triangle ABC. Find the equation of a line through the vertex B and the point P in AC; such that AP: CP = 2: 3. **[4]**

70. Find the equation of the line passing through (2, -5) and making an intercept of -3 on the Y-axis. **[4]**

71. Find the equation of a line parallel to the line $2x+y-7=0$ and passing through the intersection of the lines $x+y-4=0$ and $2x-y=8$. **[4]**

72. Find the value of **p** if the lines, 5x - 3y + 2 = 0 and 6x - py + 7 = 0 are perpendicular to each other. Hence, find the equation of a line passing through (- 2, -1) and parallel to 6x - py + 7 = 0. **[4]**

73. Find the equation of the line passing through the point $\left(9, \frac{1}{2}\right)$ and having slope $-\frac{3}{2}$. **[4]**

74. Use graph sheet to answer this question. Take 2 cm $=1$ unit along both the axes. **[5]**
   a. Plot A, B, C where A(0, 4), B(1, 1) and C(4, 0)
   b. Reflect A and B on the $x$-axis and name them as E and D respectively.
   c. Reflect $B$ through the origin and name it $F$. Write down the coordinates of $F$.
   d. Reflect B and C on the $y$-axis and name them as H and G respectively.
   e. Join points A, B, C, D, E, F, G, H and A in order and name the closed figure formed.

75. The point $P(3,4)$ is reflected to $P'$ in the $x$-axis, and $O'$ is the image of $O$ (the origin) when **[5]**

reflected in the line $PP'$.

Using a graph paper, give the

i. coordinates of $P'$ and $O'$.

ii. lengths of the segments $PP'$ and $OO'$.

iii. perimeter of the quadrilateral $POP'O'$.

iv. geometrical name of the figure $POP'O'$.

## Solution
## QUESTIONS BANK CLASS 10
## Class 10 - Mathematics
### Section A

1. **(a)** 9
**Explanation:**
Given, $3 : x :: 6 : 18$
$\therefore \frac{3}{x} = \frac{6}{18}$
$\Rightarrow 6x = 3 \times 18$
$\Rightarrow x = \frac{3 \times 18}{6}$
$\Rightarrow x = 9$
Hence, the required value of $x$ is 9.

2.
**(d)** $\frac{c+d}{c-d}$
**Explanation:**
Given, $a$, $b$, $c$ and $d$ are proportional.
Then, $\frac{a}{b} = \frac{c}{d}$
So, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ [by componendo and dividendo rule]

3.
**(c)** 3
**Explanation:**
Given, $x$, $5.4$, $5$ and 9 are in proportion, then
$\frac{x}{5.4} = \frac{5}{9}$
$\Rightarrow x = \frac{5 \times 5.4}{9}$
$= \frac{5 \times 54}{9 \times 10} = \frac{6}{2} = 3$

4. **(a)** 0.08
**Explanation:**
Let mean proportion be x.
$\Rightarrow 0.02 : x = x : 0.32$
$\Rightarrow \frac{0.02}{x} = \frac{x}{0.32}$
$\Rightarrow x^2 = 0.02 \times 0.32$
$\Rightarrow x^2 = 0.0064$
$\Rightarrow x = \sqrt{0.0064}$
$\Rightarrow x = 0.08$

5.
**(b)** 1 : 4
**Explanation:**
Given,
Mean proportion between x and y is 6.
$\Rightarrow x : 6 = 6 : y$
$\Rightarrow \frac{x}{6} = \frac{6}{y}$
$\Rightarrow y = \frac{36}{x} \ldots(1)$

Given,
The third proportional to x and y is 48.

$\Rightarrow x : y = y : 48$

$\Rightarrow \frac{x}{y} = \frac{y}{48}$

$\Rightarrow y^2 = 48x \ldots (2)$

Substitute value of y from equation 1 in equation 2

$\Rightarrow 48x = \left(\frac{36}{x}\right)^2$

$\Rightarrow 48x = \left(\frac{1296}{x^2}\right)$

$\Rightarrow 48x \times x^2 = 1296$

$\Rightarrow 48x^3 = 1296$

$\Rightarrow x^3 = \frac{1296}{48}$

$\Rightarrow x^3 = 27$

$\Rightarrow x = \sqrt[3]{27}$

$\Rightarrow x = 3$

Substituting value of x in equation 1, we get:

$\Rightarrow y = \frac{36}{3}$

$\Rightarrow y = 12$

$\Rightarrow x : y = 3 : 12$

$\Rightarrow x : y = 1 : 4.$

6. **(a)** 163

**Explanation:**

Given,

$a = 7$

$d = 10 - 7 = 3$

$a_n = 184$

We know that,

$\Rightarrow a_n = a + (n - 1)d$

$\Rightarrow a_n = 7 + (n - 1)(3)$

$\Rightarrow 184 = 7 + (n - 1)(3)$

$\Rightarrow 184 - 7 = (n - 1)(3)$

$\Rightarrow 177 = (n - 1)(3)$

$\Rightarrow \frac{177}{3} = (n - 1)$

$\Rightarrow n - 1 = 59$

$\Rightarrow n = 59 + 1$

$\Rightarrow n = 60.$

The 8th term from the end is the $(n - 8 + 1)$th term from the beginning.

$= 60 - 8 + 1$

$= 53.$

$\Rightarrow a_{53} = 7 + (53 - 1)3$

$= 7 + (52)(3)$

$= 7 + 156$

$= 163.$

7.

**(c)** 2

**Explanation:**

Let first term of G.P. be a and common ratio be r.

By formula,

$\Rightarrow a_n = ar^{n-1}$

Given,

4th term of a G.P. is 16.

$\Rightarrow a_4 = 16$

$\Rightarrow ar^{4-1} = 16$

$\Rightarrow ar^3 = 16$ ...(1)

Given,

7th term of a G.P. is 128.

$\Rightarrow a_7 = 128$

$\Rightarrow ar^{7-1} = 128$

$\Rightarrow ar^6 = 128$ ...(2)

Dividing equation (2) by (1), we get:

$\Rightarrow \frac{ar^6}{ar^3} = \frac{128}{16}$

$\Rightarrow r^3 = 8$

$\Rightarrow r^3 = 2^3$

$\Rightarrow r = 2$

8.

**(c)** $-\frac{50}{3}$

**Explanation:**

The above series is an A.P. with first term $= -\frac{2}{3}$ and common difference $= d = -\frac{2}{3} - \left(-\frac{2}{3}\right) = 0$

We know that,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore S_{25} = \frac{25}{2}\left[2 \times -\frac{2}{3} + (25-1) \times 0\right]$

$= \frac{25}{2}\left[-\frac{4}{3} + 24 \times 0\right]$

$= \frac{25}{2} \times -\frac{4}{3}$

$= -\frac{50}{3}.$

9.

**(c)** -4

**Explanation:**

In a G.P.,

$\frac{\text{any term}}{\text{preceding term}} = \text{ common ratio.}$

$\therefore \frac{2(k+1)}{k} = \frac{3(k+1)}{2(k+1)}(\text{ Eq } 1)$

$\Rightarrow (2k+2)(2k+2) = k(3k+3)$

$\Rightarrow 4k^2 + 4k + 4k + 4 = 3k^2 + 3k$

$\Rightarrow 4k^2 - 3k^2 + 8k - 3k + 4 = 0$

$\Rightarrow k^2 + 5k + 4 = 0$
$\Rightarrow k^2 + k + 4k + 4 = 0$
$\Rightarrow k(k + 1) + 4(k + 1) = 0$
$\Rightarrow (k + 1)(k + 4) =$
But $k + 1 \neq 0$ as that will not satisfy Eq 1
$\Rightarrow k + 4 = 0$
$\Rightarrow k = -4.$

10.
**(d)** y-axis
**Explanation:**
We know that,
On reflection in y-axis the sign of x-coordinate changes.
$\therefore$ Point (5, 6) is reflected in y-axis to get the point (-5, 6).

11.
**(c)** (-2, -3)
**Explanation:**
Reflection in the $x$-axis changes the sign of the $y$-coordinate only, while the $x$-coordinate remains the same.
Applying this rule to $(-2, 3)$:
$(-2, 3) \Rightarrow (-2, -3)$

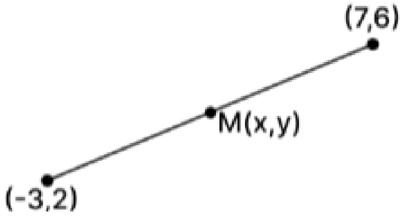
12. **(a)** (-4, -7)
**Explanation:**
Reflection in the y-axis changes the sign of the x-coordinate only, while the y-coordinate remains the same.
Applying this rule to (4, -7):
(4, -7) $\Rightarrow$ (-4, -7).

13.
**(d)** 4 units
**Explanation:**
The distance of any point $(x, y)$ from the x-axis is given by the absolute value of the $y$-coordinate, $|y|$.
For the point $P(-3, -4)$, the distance from the $x$-axis is $|-4|$.
Distance $= 4$ units.

14.
**(d)** (-5, 0)
**Explanation:**
A point lying on the x-axis has a y-coordinate of 0.
A distance of 5 units from the y-axis means the x-coordinate is $\pm 5$.
"To the left side of the origin" implies the x-coordinate is negative.
Therefore, the co-ordinates of the point are (-5, 0).

15. **(a)** $(2, 4)$
**Explanation:**

Let the mid-point be M(x, y).




By mid-point formula,

$$(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Substituting values we get:

$$\Rightarrow M(x, y) = \left(\frac{-3+7}{2}, \frac{2+6}{2}\right)$$

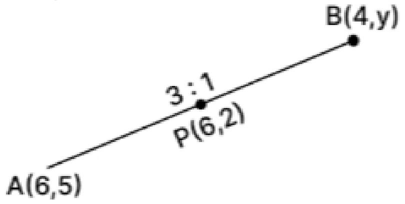
$$\Rightarrow \left(\frac{4}{2}, \frac{8}{2}\right)$$

$\Rightarrow (2, 4).$

16. **(a)** 1

**Explanation:**

Let point P be $(6, 2)$.




Given,

$m_1 : m_2 = 3 : 1$

$P(6, 2)$ divides the line segment joining $A(6, 5)$ and $B(4, y)$ in the ratio 3 : 1.

By section-formula,

$$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

We use the y -coordinate to find y.

$\Rightarrow 2 = \frac{3\times y+1\times 5}{3+1}$

$\Rightarrow 2 = \frac{3y+5}{4}$

$\Rightarrow 8 = 3y + 5$

$\Rightarrow 3y = 3$

$\Rightarrow y = 1.$

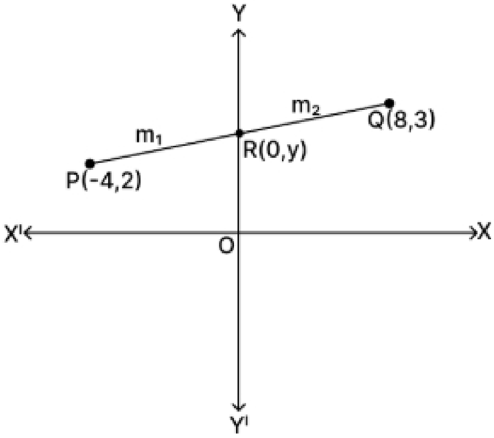
17.

**(c)** $1 : 2$

**Explanation:**

Let the point where $y$-axis divides the line segment be $R(0, y)$.

Let the ratio be $\mathrm{m}_1 : \mathrm{m}_2$.




Using section-formula,

$\Rightarrow x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$\Rightarrow 0 = \frac{m_1 \times 8 + m_2 \times (-4)}{m_1 + m_2}$

$\Rightarrow 0 = 8m_1 - 4m_2$

$\Rightarrow 8m_1 = 4m_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{4}{8} = \frac{1}{2}$

Thus, the required ratio is $1:2$.

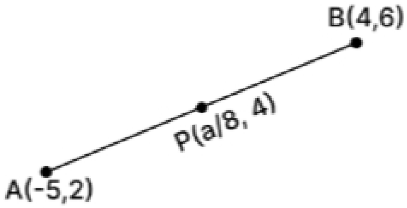
18.

**(d)** -4

**Explanation:**

Given,

$P\left(\frac{a}{8}, 4\right)$ is the mid-point of the line segment joining the points $\mathrm{A}(-5, 2)$ and $\mathrm{B}(4, 6)$.




By mid-point formula,

$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Substituting values we get:

$\Rightarrow \left(\frac{a}{8}, 4\right) = \left(\frac{-5+4}{2}, \frac{2+6}{2}\right)$

$\Rightarrow \left(\frac{a}{8}, 4\right) = \left(\frac{-1}{2}, 4\right)$

Comparing the x -coordinates, we get:

$\Rightarrow \frac{a}{8} = \frac{-1}{2}$

$\Rightarrow a = \frac{-8}{2}$

$\Rightarrow a = -4$

19.

**(c)** $-\frac{4}{3}$

**Explanation:**

Given, $3x = 4y + 11$.

$\Rightarrow 4y = 3x - 11$

$\Rightarrow y = \frac{3}{4}x - \frac{11}{4}$.

Comparing with $y = mx + c$ we get,

Slope $\left(m_1\right) = \frac{3}{4}$.

Let the slope of perpendicular line be $m_2$. Since, lines are perpendicular so,

$\Rightarrow m_1 \times m_2 = -1$

$\Rightarrow \frac{3}{4} \times m_2 = -1$

$\Rightarrow m_2 = -\frac{4}{3}$

20.

**(c)** $y = 3x + 1$

**Explanation:**

By intercept form :

Equation of line : $\frac{x}{a} + \frac{y}{b} = 1$ ...(1)

Given,

Line cuts equal intercepts with both the axes.

∴ x -intercept $(a)$ = $y$-intercept $(b)$ = $p$ (let)

Given,

Line passes through point (6, 6). So it will satisfy the equation (1).

Substituting values in equation we get:

$\Rightarrow \frac{6}{p} + \frac{6}{p} = 1$

$\Rightarrow \frac{12}{p} = 1$

$\Rightarrow p = 12$.

$\therefore a = b = 12$.

Substituting value of a and b in equation (1), we get:

$\Rightarrow \frac{x}{12} + \frac{y}{12} = 1$

$\Rightarrow \frac{x+y}{12} = 1$

$\Rightarrow x + y = 12$.

21.

**(b)** $x - y = 7$

**Explanation:**

By intercept form,

Equation of line: $\frac{x}{a} + \frac{y}{b} = 1$

Given,

x-intercept (a): 7

y-intercept (b): -7

Substituting values in equation, we get:

$\Rightarrow \frac{x}{7} + \frac{y}{-7} = 1$

$\Rightarrow \frac{x}{7} - \frac{y}{7} = 1$

$\Rightarrow x - y = 7.$

22.
**(d)** $2x + y = 3$
**Explanation:**
By slope-intercept form :
Equation of line : y = mx + c ...(1)
Given,
Line passes through point (1, 1) and makes a y-intercept of 3.
Substituting value in equation (1), we get :
⇒ 1 = m(1) + 3
⇒ 1 = m + 3
⇒ m = 1 - 3 = -2.
∴ Slope of line (m) = -2 and y-intercept (c) = 3
Substituting value of m and c in equation (1), we get :
⇒ y = -2x + 3
⇒ 2x + y = 3.

**Section B**

23. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
We know that if four quantities $a, b, c$ and $d$ form a proportion, then
$\frac{a}{b} = \frac{c}{d}$
$\therefore$ If $3 : x :: 2 : 18$, then we get
$\frac{3}{x} = \frac{2}{18} \Rightarrow \frac{3}{x} = \frac{1}{9}$
$\Rightarrow x = 27$
Hence, Both A and R are true and R is the correct explanation of A.

24. **(a)** Both A and R are true and R is the correct explanation of A.
**Explanation:**
Let first n natural numbers: $1, 2, 3, \ldots, n$.
The above sequence is an A.P. with first term = 1, common difference $= 1$ and number of terms $= n$.
By formula,
Sum of an A.P. $= \frac{n}{2}[2a + (n - 1)d]$
Substituting values we get :
$S = \frac{n}{2} \times [2 \times 1 + (n - 1) \times 1]$
$= \frac{n}{2} \times [2 + n - 1]$
$= \frac{n}{2} \times (n + 1).$
So, reason (R) is true.
When $n = 99$.
The sum of first 99 natural numbers
$= \frac{99(99+1)}{2}$
$= \frac{99 \times 100}{2}$
$= \frac{9900}{2}$
$= 4950$
So, assertion (A) is true and, reason (R) is the correct explanation of assertion (A).

Thus, both Assertion (A) and Reason (R) are correct, and Reason (R) is the correct reason for Assertion (A).

25.

**(c)** A is true but R is false.

**Explanation:**

Given, vertices of the $\triangle ABC$ are

$A(-1, 3), B(1, -1)$ and $C(5, 1)$.

Let the coordinates of centroid be $G(x, y)$.

Then, $x = \frac{-1+1+5}{3} = \frac{5}{3}$ and $y = \frac{3-1+1}{3} = \frac{3}{3} = 1$

[ $\because$ coordinates of centroid $= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$]

Hence, the coordinates of the centroid are $\left(\frac{5}{3}, 1\right)$.

Hence, $A$ is true, $R$ is false.

26. **(a)** Both A and R are true

**Explanation:**

Slope $(m_1) = \frac{y_2-y_1}{x_2-x_1} = \frac{6-5}{-3-2} = -\frac{1}{5}$

The product of the slopes of two perpendicular lines is always -1. Let slope of perpendicular line be $m_2$.

$\Rightarrow m_1 \times m_2 = -1$

$\Rightarrow -\frac{1}{5} \times m_2 = -1$

$\Rightarrow m_2 = 5$

Both A and R are true

## Section C

27.

**(d)** Statement I is true and Statement II is false.

**Explanation:**

**Statement I:** Given, $a$, 4, 12 and $b$ are in continued proportion.

$\therefore \frac{a}{4} = \frac{4}{12} = \frac{12}{b}$

$\Rightarrow \frac{a}{4} = \frac{4}{12}$ and $\frac{4}{12} = \frac{12}{b}$

$\Rightarrow a = \frac{16}{12}$ and $b = \frac{12 \times 12}{4}$

$\Rightarrow a = \frac{4}{3}$ and $b = 36$

$\therefore$ Statement I is true.

**Statement II:** We have, $\frac{a^2+b^2}{a^2-b^2} = \frac{17}{8}$

Using componendo and dividendo property, we get

$\frac{a^2+b^2+a^2-b^2}{a^2+b^2-a^2+b^2} = \frac{17+8}{17-8} \Rightarrow \frac{2a^2}{2b^2} = \frac{25}{9}$

$\Rightarrow \frac{a}{b} = \frac{5}{3}$ i.e. $a:b = 5:3$

$\therefore$ Statement II is false.

28.

**(d)** Both the Statements are true.

**Explanation:**
**Statement** I: Let $A$ be the first term and $R$ be the common ratio of the given GP.
Now,
$a_p = AR^{p-1}$,
$a_q = AR^{q-1}$
$a_r = AR^{r-1}$
and
Since, $a_p$, $a_q$ and $a_r$ are in GP.

$$\therefore \left(a_q\right)^2 = \left(a_p\right)\left(a_r\right)$$
$$\Rightarrow \left(AR^{q-1}\right)^2 = \left(AR^{p-1}\right)\left(AR^{r-1}\right)$$
$$\Rightarrow A^2 \cdot R^{2q-2} = A^2R^{p+r-2}$$
$$\Rightarrow 2q - 2 = p + r - 2$$
$$\Rightarrow 2q = p + r$$

Thus, $p$, $q$ and $r$ are in AP.
So, Statement I is true.
**Statement II:** In GP a, ar, ar $^2$, …, if $r = 1$, then
$S_n = a + a + a + \ldots + n \text{ terms } = na$
Thus, Statement II is true.

29.
**(d)** Both the Statements are false.
**Explanation:**
**Statement I:** We know that if $P(x, y)$ is any point in the coordinate plane, then the point $P'(-x, -y)$ is the reflection of the point $P$ in the origin.
Thus, Statement I is false.
**Statement II:** We know that the reflection of the point $P(x, y)$ in the line $x = a$ is $P'(-x + 2a, y)$.
Hence, Statement II is also false.

30.
**(b)** Both the Statements are true.
**Explanation:**
**Statement I:** Given, vertices of a triangle are (6, 4), $(3, 2)$ and $(1, 6)$.
As we know that the intersection point of medians in a triangle is called centroid of the triangle.
Therefore, the coordinates of intersection point of medians

$$= \left(\frac{6+3+1}{3}, \frac{4+2+6}{3}\right)$$

$$\left[\begin{array}{l} \because \text{ the coordinates of centroid of a triangle} \\ \qquad = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) \end{array}\right]$$

$$= \left(\frac{10}{3}, 4\right)$$

$\because$ Statement I is true.
**Statement II:** Given, vertices of a triangle are (6, 11), $(8, 3)$ and $(x, 4)$.

Now, centroid of the triangle $= \left(\frac{6+8+x}{3}, \frac{11+3+4}{3}\right)$

$\Rightarrow (9, 6) = \left(\frac{14+x}{3}, 6\right)$ [given, centroid of triangle is $(9, 6)$]

On comparing $x$-coordinates both sides, we get

$9 = \frac{14+x}{3} \Rightarrow x = 27 - 14 \Rightarrow x = 13$

Hence, Statement II is also true.

31. **(a)** Statement I is true and Statement II is false.

**Explanation:**

**Statement I:** Given, the two points $A$ and $B$ are $(-2, 3)$ and $(2, 7)$, respectively.

$\therefore$ Slope of $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{7-3}{2+2} = \frac{4}{4} = 1$

Let the inclination of line $AB$ be $\theta$.

Then, $\tan\theta = 1$[ $\because$ slope of $AB = 1$]

$\Rightarrow \tan\theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$

**Statement II:** Given, the two points $A$ and $B$ are $(2, 3)$ and $(-5, 0)$, respectively.

$\therefore$ Slope of $AB = \frac{y_2-y_1}{x_2-x_1} = \frac{0-3}{-5-2} = \frac{3}{7}$

Let the slopes be $m_1$ and $m_2$.

Since, two lines are parallel.

$\therefore m_1 = m_2$

So, the slope of the line parallel to $AB$ is $\frac{3}{7}$.

Hence, Statement I is true and Statement II is false.

## Section D

32. Given $a, b$ and $c$ are in continued proportion, then

$\frac{a}{b} = \frac{b}{c} \Rightarrow ac = b^2$

Now,

LHS $= (a + b + c)(a - b + c)$

$= a^2 + ab + ac - ab - b^2 - bc + ac + bc + c^2$

$= a^2 - b^2 + c^2 + 2ac$

$= a^2 - b^2 + c^2 + 2b^2\left[\because ac = b^2\right]$

$= a^2 + b^2 + c^2 =$ RHS

Hence proved.

33. a, b, c, d are in proportion

$\therefore \frac{a}{b} = \frac{c}{d} = k$

Hence, $a = bk, c = dk$.

L.H.S. $= \left(a^4 + c^4\right) : \left(b^4 + d^4\right)$

$= \frac{a^4+c^4}{b^4+d^4}$

$= \frac{k^4b^4+k^4d^4}{b^4+d^4}$

$= k^4$.

R.H.S. $= a^2c^2 : b^2d^2$

$= \frac{a^2c^2}{b^2d^2}$

$= \frac{(bk)^2(dk)^2}{b^2d^2}$

$= \dfrac{k^4 b^2 d^2}{b^2 d^2}$

$= k^4.$

Since, L.H.S. = R.H.S.

Hence proved.

34. Given, $(4a + 5b)(4c - 5d) = (4a - 5b)(4c + 5d)$.

On cross-multiplication,

$\Rightarrow \dfrac{4a+5b}{4a-5b} = \dfrac{4c+5d}{4c-5d}$

By componendo and dividendo,

$\Rightarrow \dfrac{4a+5b+4a-5b}{4a+5b-4a+5b} = \dfrac{4c+5d+4c-5d}{4c+5d-4c+5d}$

$\Rightarrow \dfrac{8a}{10b} = \dfrac{8c}{10d}$

On dividing the equation by $\dfrac{8}{10}$,

$\Rightarrow \dfrac{a}{b} = \dfrac{c}{d}$

$\Rightarrow a : b :: c : d.$

Hence, proved that $a, b, c, d$ are in proportion.

35. Given,

$\dfrac{\sqrt{x+4}+\sqrt{x-10}}{\sqrt{x+4}-\sqrt{x-10}} = \dfrac{5}{2}$

Applying componendo and dividendo,

$\Rightarrow \dfrac{\sqrt{x+4}+\sqrt{x-10}+\sqrt{x+4}-\sqrt{x-10}}{\sqrt{x+4}+\sqrt{x-10}-\sqrt{x-4}+\sqrt{x-10}} = \dfrac{5+2}{5-2}$

$\Rightarrow \dfrac{2\sqrt{x+4}}{2\sqrt{x-10}} = \dfrac{7}{3}$

$\Rightarrow \dfrac{\sqrt{x+4}}{\sqrt{x-10}} = \dfrac{7}{3}$

Squaring both sides,

$\Rightarrow \dfrac{x+4}{x-10} = \dfrac{49}{9}$

$\Rightarrow 9(x + 4) = 49(x - 10)$

$\Rightarrow 9x + 36 = 49x - 490$

$\Rightarrow 40x = 526$

$\Rightarrow x = \dfrac{526}{40}$

$\Rightarrow x = \dfrac{263}{20}$

Hence, the value of $x = \dfrac{263}{20}$.

36. Reversing the order.

185, 181, ... 13, 9, 5

a = 185, d = -4, n = 9

$\therefore$ $a_9$ = a + (9 - 1)d

= 185 + 8(-4)

= 185 - 32

= 153

37. Given, geometric progression is $7, 21, 63, \ldots$

Here, first term, $a = 7$

Common ratio, $r = \dfrac{21}{7} = 3$

Also given, $n$th term of GP $= 1701$

i. $\therefore T_n = a \cdot r^{n-1} = 1701$

$\Rightarrow 7(3)^{n-1} = 1701$

$\Rightarrow (3)^{n-1} = \frac{1701}{7} = 243$

$\Rightarrow (3)^{n-1} = (3)^5$

$\therefore n - 1 = 5 \Rightarrow n = 6$

ii. We know that

Sum of $n$ terms of GP,

$S_n = \frac{a\left(r^n - 1\right)}{r - 1}, r > 1$

$\Rightarrow S_n = \frac{7(729-1)}{3-1} \Rightarrow S_n = \frac{7(229-1)}{2}$

$\Rightarrow S_n = \frac{7 \times 728}{2} = 7 \times 364 = 2548$

Hence, the sum of $n$ terms of the given GP is 2548.

38. Here, $a = a_1 = n - 2, a_2 = 4n - 1$ and $a_3 = 5n + 2$

Since, the numbers are in A.P. so,

$a_2 - a_1 = d = a_3 - a_2$

$\Rightarrow a_2 - a_1 = a_3 - a_2$

$\Rightarrow 4n - 1 - (n - 2) = 5n + 2 - (4n - 1)$

$\Rightarrow 4n - n - 1 + 2 = 5n - 4n + 2 + 1$

$\Rightarrow 3n + 1 = n + 3$

$\Rightarrow 3n - n = 3 - 1$

$\Rightarrow 2n = 2$

$\Rightarrow n = 1$

Hence, the value of $n = 1$.

39. Given,

Common ratio $(r) = 2$

By formula,

$\Rightarrow a_n = ar^{n-1}$

Given,

$10^{\text{th}}$ term of G.P. $= 768$

$\Rightarrow a_{10} = 768$

$\Rightarrow ar^{10-1} = 768$

$\Rightarrow ar^9 = 768$

$\Rightarrow a \times 2^9 = 768$

$\Rightarrow a \times 512 = 768$

$\Rightarrow a = \frac{768}{512} = \frac{3}{2}$

8th term of G.P. is given by:

$\Rightarrow a_8 = ar^{8-1}$

$\Rightarrow a_8 = ar^7$

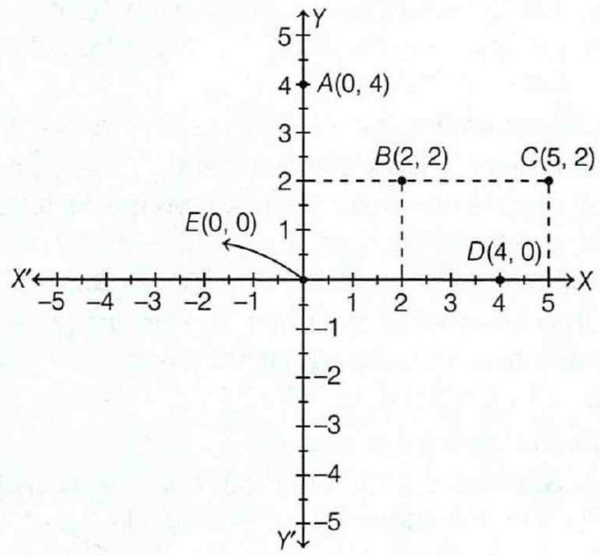
$\Rightarrow a_8 = \frac{3}{2} \times 2^7$

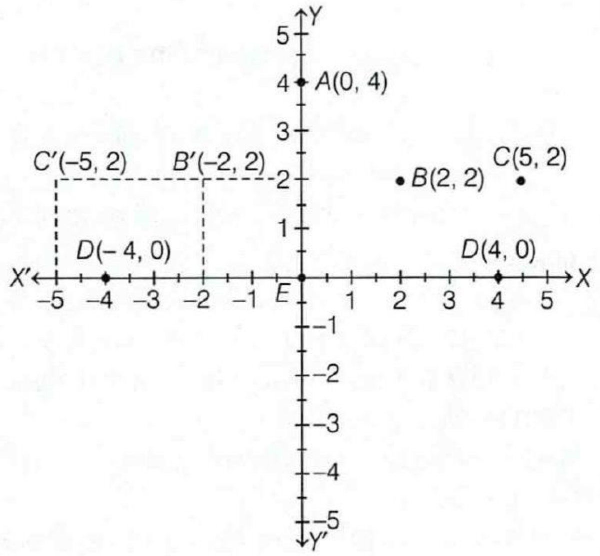
$\Rightarrow a_8 = 3 \times 2^6$

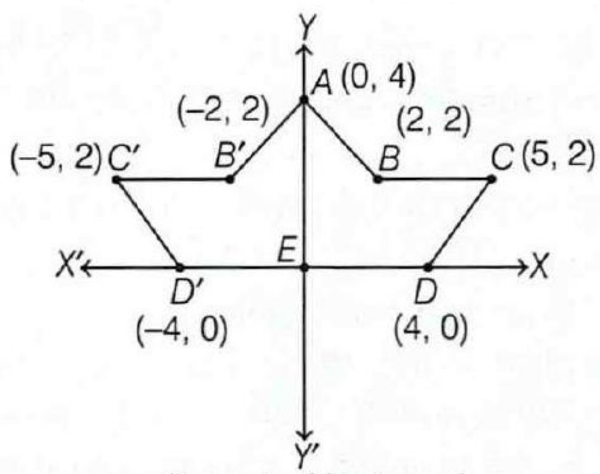
$\Rightarrow a_8 = 3 \times 64 = 192$

Hence, $8^{\text{th}}$ term of a G.P. is given by 192.

40. Let $XEX'$ and $YEY'$ are two perpendicular axes meet at origin on a graph paper and mark $2\ cm = 1$ unit on both coordinate axes.

i. 


ii. 


iii. 


(Irregular Heptagon)

41. i. We know that $A^{'}(x, -y)$ is the reflection of $A(x, y)$ in the $X$-axis.

Point $P(2, -3)$ on reflection becomes $P^{'}(2, 3)$ that means the line of reflection $\left(L_1\right)$ is $X$-axis.

ii. We know that origin remains invariant on reflection along both the axes.
Given, $L_2$ is perpendicular to $L_1$.
It means $L_2$ is perpendicular to $X$-axis and passes through $(0, 0)$ i.e. $L_2$ is $Y$-axis.
$P^{'}(2, 3)$ on reflection in $Y$-axis becomes $P^{''}(-2, 3)$.

iii. The point of intersection of $L_1$ (X-axis) and $L_2$($Y$-axis) is the origin i.e. $(0, 0)$

iv. Reflecting point $P(2, -3)$ through the origin gives us $P^{'''}(-2, 3)$ and $P^{''}$ and $P^{'''}$ have similar coordinates.

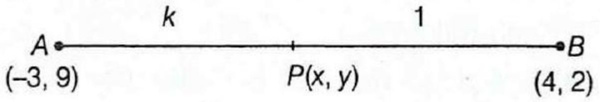
42. i. $A(-6, 7)$ and $B(3, 5)$

Mid-point of $AB$ = $\left(\frac{-6+3}{2}, \frac{7+5}{2}\right) = \left(\frac{-3}{2}, 6\right)$

ii. $A(5, -3)$ and $B(-1, 7)$

Mid-point of $AB$ = $\left(\frac{5-1}{2}, \frac{-3+7}{2}\right) = (2, 2)$

43. Let the given line divide $AB$ in the ratio $k:1$




$\therefore x = \frac{4k+(-3)}{k+1}; y = \frac{2k+9}{k+1}$ [by section formula]

Clearly, $y = 2 + 3x$

$\Rightarrow \frac{2k+9}{k+1} = 2 + \frac{3(4k-3)}{k+1}$

$\Rightarrow 2k + 9 = 2k + 2 + 12k - 9$

$\Rightarrow 12k = 16$

$k = \frac{16}{12} = \frac{4}{3}$

$\therefore$ Required ratio is $4:3$.

44. Mid-point of $AB = (2, 3)$

$\therefore \left(\frac{3+x}{2}, \frac{5+y}{2}\right) = (2, 3)$

$\Rightarrow \frac{3+x}{2} = 2$ and $\frac{5+y}{2} = 3$

$\Rightarrow 3 + x = 4$ and $5 + y = 6$

$\Rightarrow x = 1$ and $y = 1$

45. Let C(x, y) be the mid-point of the line joining the points A(3, - 1) and B (7, 9).

$\therefore$ Coordinates of mid-point of AB, i.e.

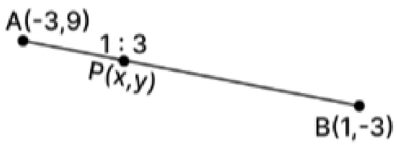
$C(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Here, $x_1 = 3$, $x_2 = 7$ and $y_1 = -1$, $y_2 = 9$

$\therefore$ $C(x, y) = \left(\frac{3+7}{2}, \frac{-1+9}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$

Hence, the mid-points of line segment joining (3, - 1) and (7, 9) is (5, 4).

46. Let Point $P$ divides the line segment joining $A(-3, 9)$ and $B(1, -3)$ internally in the ratio $m_1:m_2 = 1:3$.




By using section formula,

$P(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

Substituting values we get:

$P = \left(\frac{1\times1+3\times-3}{1+3}, \frac{1\times-3+3\times9}{1+3}\right)$

$= \left(\frac{1-9}{4}, \frac{-3+27}{4}\right)$

$= \left(\frac{-8}{4}, \frac{24}{4}\right)$

$= (-2, 6)$.

Hence, coordinates of $P = (-2, 6)$.

47. We know that the equation of straight line parallel to $x$-axis is

$y = a$

Since the line passes through the point $(-3, 2)$, we get

$a = 2$

∴ Equation of the line

$\Rightarrow y = 2$ or $y - 2 = 0$.

Hence, equation of the line is $y = 2$.

48. i. Gradient of a line $= \frac{y_2 - y_1}{x_2 - x_1}$

Putting values in above formula we get,

$\text{Gradient} = \frac{4-(-2)}{3-0}$

$= \frac{6}{3}$

$= 2$

Hence, the gradient of the line passing through $(0, -2), (3, 4)$ is 2 .

ii. Gradient of a line $= \frac{y_2 - y_1}{x_2 - x_1}$

Putting values in above formula we get,

$\text{Gradient} = \frac{8-(-7)}{-1-3}$

$= \frac{15}{-4}$

$= -\frac{15}{4}$

Hence, the gradient of the line passing through $(3, -7), (-1, 8)$ is $-\frac{15}{4}$.

49. We know that the equation of straight line parallel to y-axis is x = a.

Since the line passes through the point (-7, 5), we get

a = -7

∴ Equation of the line

⇒ x = -7 or x + 7 = 0.

Hence, equation of the line is x + 7 = 0.

50. i. The equation of the straight line is given by, $y = mx + c$, we get where m is the slope and c is the y -intercept.

Given slope $= \frac{3}{4}$ and y -intercept $= -4$. Substituting values in equation we get,

$\Rightarrow y = \frac{3}{4}x - 4$

$\Rightarrow y = \frac{3x-16}{4}$

$\Rightarrow 4y = 3x - 16$

$\Rightarrow 3x - 4y = 16$

Hence, equation of the line is $3x - 4y = 16$.

ii. The equation of straight line is given by $y = mx + c$, where m is the slope and c is the $y$-intercept.

Given slope $= \sqrt{3}$ and y -intercept $= \frac{-2}{3}$.

Substituting values in equation we get,

$\Rightarrow y = \sqrt{3}x + \left(\frac{-2}{3}\right)$

$\Rightarrow y = \frac{3\sqrt{3}x - 2}{3}$

$\Rightarrow 3y = 3\sqrt{3}x - 2$

$\Rightarrow 3\sqrt{3}x - 3y - 2 = 0$

Hence, equation of the line is $3\sqrt{3}x - 3y - 2 = 0$.

51. Given $A(2a + 2, 3)$, $B(7, 4)$ and $C(2a + 5, 2)$ are collinear.

When three points are collinear, then

$\frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right] = 0$

or $x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right) = 0$

We have

$x_1 = 2a + 2, x_2 = 7, x_3 = 2a + 5$

$y_1 = 3, y_2 = 4, y_3 = 2$

$\Rightarrow [(2a + 2)(4 - 2) + 7(2 - 3) + (2a + 5)(3 - 4)] = 0$

$4a + 4 - 7 - 2a - 5 = 0$

$2a - 8 = 0$

$a = \frac{8}{2} = 4$

52. Since, a, b, c, d are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$

$\therefore c = dk, b = ck = (dk)k = dk^2$ and $a = bk = (ck)k = (dk)k^2 = dk^3$.

L.H.S. $= (a + d)(b + c) - (a + c)(b + d)$

$= \left(dk^3 + d\right)\left(dk^2 + dk\right) - \left(dk^3 + dk\right)\left(dk^2 + d\right)$

$= d\left(k^3 + 1\right)dk(k + 1) - dk\left(k^2 + 1\right)d\left(k^2 + 1\right)$

$= d^2k\left[\left(k^3 + 1\right)(k + 1) - \left(k^2 + 1\right)^2\right]$

$= d^2k[(k^4 + k^3 + k + 1) - (k^4 + 1 + 2k^2)]$

$= d^2k(k^4 - k^4 + k^3 - 2k^2 + k + 1 - 1)$

$= d^2k\left(k^3 - 2k^2 + k\right)$

$= d^2k\left(k\left(k^2 - 2k + 1\right)\right)$

$= d^2k^2\left(k^2 - 2k + 1\right)$

$= d^2k^2(k - 1)^2$.

R.H.S. $= (b - c)^2$

$= \left(dk^2 - dk\right)^2$

$= \left[(dk)^2(k - 1)^2\right]$

$= d^2k^2(k - 1)^2$.

Since, L.H.S = R.H.S, hence proved that,

$(a + d)(b + c) - (a + c)(b + d) = (b - c)^2$.

53. We have,

$\frac{3x - 9}{5x + 4} = \frac{3^3}{4^3}$

$\Rightarrow \frac{3x-9}{5x+4} = \frac{27}{64}$

$\Rightarrow \frac{3(x-3)}{5x+4} = \frac{27}{64}$

$\Rightarrow \frac{x-3}{5x+4} = \frac{9}{64}$

$\Rightarrow 64x - 192 = 45x + 36$

$\Rightarrow 19x = 228$

$\Rightarrow x = 12$

54. Given, q is the mean proportional between p and r.

$\Rightarrow q^2 = pr$

L.H.S. $= pqr(p + q + r)^3$

$= qq^2(p + q + r)^3$

$= q^3(p + q + r)^3 \ldots \left[\because q^2 = pr\right]$

$= [q(p + q + r)]^3$

$= (pq + q + r)]^3$

$= (pq + pr + qr)^3 \left[\because q^2 = pr\right]$

= R.H.S.

55. Given, $x + 5$ is the mean proportional between $x + 2$ and $x + 9$.

$\Rightarrow$ $(x + 2)$, $(x + 5)$ and ( $x + 9$ ) are in continued proportion.

$\Rightarrow (x + 2) : (x + 5) = (x + 5) : (x + 9)$

$\Rightarrow (x + 5)^2 = (x + 2)(x + 9)$

$\Rightarrow x^2 + 25 + 10x = x^2 + 2x + 9x + 18$

$\Rightarrow 25 - 18 = 11x - 10x$

$\Rightarrow x = 7$

56. Let the terms be a - d, a, a + d

$\therefore$ a - d + a + a - d = 42

3a = 42

$\therefore$ a = 14

(a - d)(a + d) = 52

$14^2 - d^2 = 52$

$d^2 = 196 - 52$

$d^2 = 144$

$\therefore$ d = $\pm$ 12

d = 12, or -12 N.P.

First term is 14 - 12 = 2 or 14 + 12 = 26.

57. Let a be the first term and d be the common difference of an A.P.,

then 4th term = 22

i.e., a + (4 - 1) d = 22

or a + 3d = 22 ...(i)

and 15th term = 66

i.e., a + (15 - 1) d = 66

or a + 14d = 66 ...(ii)

Subtracting equation (i) from (ii), we get

11d = 44

$\Rightarrow$ d = 4

Putting the value of d in eq. (i),

a + 3 $\times$ 4 = 22

a = 10

Hence, a = 10 and d = 4

The sum of series of 8 terms,

$S_8 = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{8}{2}[2 \times 10 + (8 - 1)4]$

$= 4(20 + 28)$

$= 4 \times 48$

$= 192$

58. Let a be the first term and r the common ratio and

$T_4 = 16, T_7 = 128$ (given)

$\therefore ar^{4 - 1} = 16$ and $ar^{7 - 1} = 128$

$ar^3 = 16$ ...(i)

$ar^6 = 128$ ...(ii)

Dividing eq. (iii) by (i),

$\frac{ar^6}{ar^3} = \frac{128}{16}$

$\therefore r^3 = 8 = 2^3$

$\therefore r = 2$

$ar^3 = 16 \Rightarrow a \times 8 = 160$

$\therefore a = 2$

Hence, first term is 2 and the common ratio is 2.

59. Let the first term of given A.P. be a and common difference be d.

$\because T_n = a + (n - 1)d$

$\therefore T_2 = a + (2 - 1)d = 10$

$\Rightarrow a + d = 10$ ...(i)

and $T_{45} = a + (45 - 1)d$

$\Rightarrow a + 44d = 96$ ...(ii)

On subtracting (i) from (ii), we get

$43d = 86$

$d = \frac{86}{43} = 2$

Substitution this value of d in eq. (i)

$a + 2 = 10$

$a = 10 - 2 = 8$

So first term of A.P. (a) = 8 and common difference (d) = 2.

$\because S_n = \frac{n}{2}[2a + (n - 1)d]$

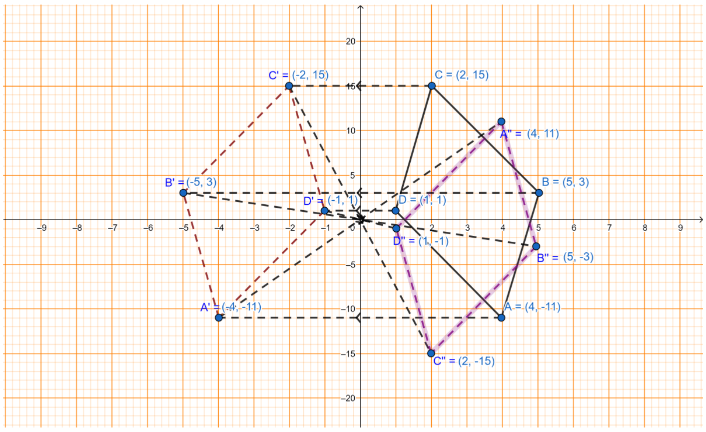
$\therefore S_{15} = \frac{15}{2}[2 \times 8 + (15 - 1)2]$

$= \frac{15}{2}[16 + 28]$

$S_{15} = 330$

Hence, sum of 15 terms = 330

60. The graph for this problem is shown below:




First the parallelogram is reflected in y-axis.
We know that,
Rule to find reflection of a point in y-axis:

1. Change the sign of abscissa i.e. x-coordinate.
2. Retain the ordinate i.e. y-coordinate.

∴ Coordinates of
⇒ A(4, -11) on reflection in y-axis becomes A'(-4, -11).
⇒ B(5, 3) on reflection in y-axis becomes B'(-5, 3).
⇒ C(2, 15) on reflection in y-axis becomes C'(-2, 15).
⇒ D(1, 1) on reflection in y-axis becomes D'(-1, 1).
We know that,
Rules to find the reflection of a point in the origin:

1. Change the sign of abscissa i.e. x-coordinate.
2. Change the sign of ordinate i.e. y-coordinate.

∴ Coordinates of
⇒ $A'(-4, -11)$ on reflection in origin becomes $A''(4, 11)$.
⇒ $B'(-5, 3)$ on reflection in origin becomes $B''(5, -3)$.
⇒ $C'(-2, 15)$ on reflection in origin becomes $C''(2, -15)$.
⇒ $D'(-1, 1)$ on reflection in origin becomes $D''(1, -1)$
From graph we can see that A"B"C"D" is also a parallelogram.
The single transformation that marks the following changes i.e.
$A(4, -11) \Rightarrow A''(4, 11)$
$B(5, 3) \Rightarrow B''(5, -3)$
$C(2, 15) \Rightarrow C''(2, -15)$
$D(1, 1) \Rightarrow D''(1, -1)$, is reflection in $x$-axis.
**Hence,**

1. **The coordinates of the final images of the vertices are (4, 11), (5, -3), (2, -15) and (1, -1) respectively.**
2. **These new images still form parallelogram.**
3. **The single transformation from ABCD ⇒ A"B"C"D" can be achieved by reflection in x-axis.**

61. Given, A(1, -3) and B(2, 4)
    i. ∵ x = 0, It represent the y-axis.
    So, A' is the image of A(1, -3) across y-axis
    Thus co-ordinates of A' will be (-1, -3)
    ii. Equation of line AA' will be

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-3) = \frac{-3-(-3)}{-1-1}(x - 1)$$

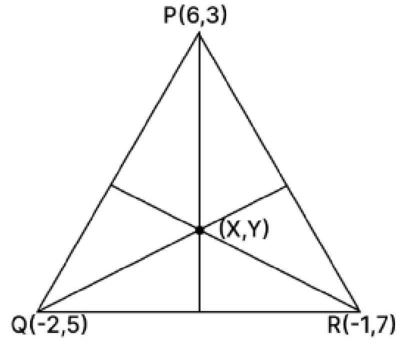
$$y + 3 = \frac{0}{-2}(x - 1)$$

$y + 3 = 0(x - 1)$
$y = -3$
B' is the image of point B across the line y = -3 will be (2, -10)

62. By using the centroid formula,

$$(x, y) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$P(6, 3)$, $Q(-2, 5)$ and $R(-1, 7)$




Substitute values we get:

$$\Rightarrow (x, y) = \left(\frac{6+(-2)+(-1)}{3}, \frac{3+5+7}{3}\right)$$

$$\Rightarrow (x, y) = \left(\frac{6-3}{3}, \frac{15}{3}\right)$$

$$\Rightarrow (x, y) = \left(\frac{3}{3}, \frac{15}{3}\right)$$

$\Rightarrow (x, y) = (1, 5).$

Hence, coordinates of centroid of $\triangle \text{PQR} = (1, 5).$

63. It is given that the mid-point of the line-segment joining $(4a, 2b-3)$ and $(-4, 3b)$ is $(2, -2a)$.

$$\therefore (2, -2a) = \left(\frac{4a-4}{2}, \frac{2b-3+3b}{2}\right)$$

$$\Rightarrow 2 = \frac{4a-4}{2}$$

$\Rightarrow 4a - 4 = 4$
$\Rightarrow 4a = 8$
$\Rightarrow a = 2$

Also,

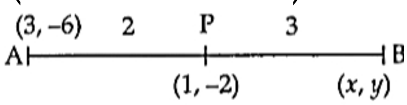
$$-2a = \frac{2b-3+3b}{2}$$

$$\Rightarrow -2 \times 2 = \frac{5b-3}{2}$$

$\Rightarrow 5b - 3 = -8$
$\Rightarrow 5b = -5$
$\Rightarrow b = -1$

64. Applying section formula

$$\left(\frac{(3\times3)+2x}{2+3}, \frac{(3\times-6)+2y}{2+3}\right) = P$$




$$\therefore \frac{9+2x}{5} = 1$$

$\therefore$ 9 + 2x = 5

$\therefore$ 2x = -4

$\therefore$ x = -2

$\frac{-18+2y}{5}$ = -2

$\therefore$ -18 + 2y = -10

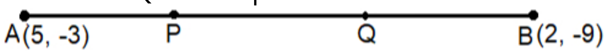
$\therefore$ 2y = 8

$\therefore$ y = 4

$\therefore$ B = (-2, 4)

65. Let the points be A(5, -3) and B(2, -9).

Let P and Q be the points of trisection of AB.




P divides AB in the ratio 1 : 2

By section formula, $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

the coordinates of P are

$\left(\frac{1\times 2+2\times 5}{1+2}, \frac{1\times(-9)+2\times(-3)}{1+2}\right) = \left(\frac{2+10}{3}, \frac{-9-6}{3}\right)$ = (4, -5)

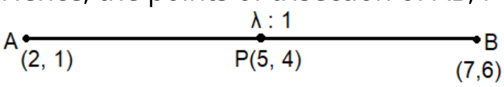
P(4, -5)

Q divides AB in the ratio 2 : 1, then the coordinates of Q are

$\left(\frac{2\times 2+1\times 5}{2+1}, \frac{2\times(-9)+1\times(-3)}{2+1}\right) = \left(\frac{9+5}{3}, \frac{-18-3}{3}\right)$ = (3, -7)

Q(3, -7)

Hence, the points of trisection of AB, P and Q P(4, -5) and Q(3, -7).

66. 


Given, the Point A(2, 1) and B(7, 6).

Let point P divides the AB in the ratio $\lambda$ : 1.

So, the x-coordinate of P is

$\frac{mx_2+nx_1}{m+n} = 5$

$\Rightarrow \frac{\lambda\times 7+1\times 2}{\lambda+1} = 5$

$\Rightarrow \frac{7\lambda+2}{\lambda+1} = 5$

$\Rightarrow$ $7\lambda + 2 = 5\lambda + 5$

$2\lambda = 3$

$\therefore \lambda = \frac{3}{2}$

Hence, Ratio is 3 : 2.

67. Let the third vertex be C($x_3$, $y_3$)

$x_1 = 3$, $y_1 = -5$

$x_2 = -7$, $y_2 = 4$

Centroid = $\left[\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right]$

$2 = \frac{3+(-7)+x_3}{3}$

$x_3 = 6 + 4 = 10$

$\therefore$ $x_3 = 10$

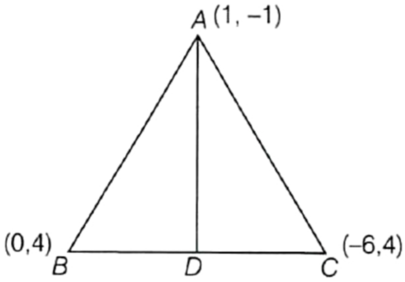
$-1 = \frac{y_1+y_2+y_3}{3}$

-3 = -5 + 4 + $y_3$

$\therefore$ $y_3$ = -3 + 1 = -2

Hence the third vertex be (10, -2).

68. Given, vertices of $\triangle ABC$ are $A(1, -1)$, $B(0, 4)$ and $C(-6, 4)$ and $D$ is the mid-point of $BC$.




i. Mid-point of $BC = \left(\frac{0-6}{2}, \frac{4+4}{2}\right) = (-3, 4)$

$$\left[\because \text{ mid - point } = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$$

$\therefore$ Coordinates of $D$ are $(-3, 4)$.

ii. Equation of the line passing through two points $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ is given by

$$y - y_1 = \frac{y_2-y_1}{x_2-x_1}\left(x - x_1\right)$$

$$\Rightarrow \frac{y-y_1}{y_2-y_1} = \frac{\left(x-x_1\right)}{\left(x_2-x_1\right)}$$

$\therefore$ Equation of the median $AD$ having $A(1, -1)$ and $D(-3, 4)$ is given by

$$\Rightarrow \frac{y-(-1)}{4-(-1)} = \frac{x-1}{-4}\left[\begin{array}{l} \because x_1 = 1 \text{ and } y_1 = -1 \\ x_2 = -3 \text{ and } y_2 = 4 \end{array}\right]$$

$$\Rightarrow -4y - 4 = 5x - 5 \Rightarrow 5x + 4y = 1$$

Hence, the equation of median $AD$ is $5x + 4y = 1$.

69. Given, $AP : CP = 2 : 3$

$\therefore$ Co-ordinates of P are

$$\left(\frac{2\times(-3)+3\times 7}{2+3}, \frac{2\times 4+3\times(-1)}{2+3}\right)$$

$$= \left(\frac{-6+21}{5}, \frac{8-3}{5}\right)$$

$$= \left(\frac{15}{5}, \frac{5}{5}\right)$$

$= (3, 1)$

Slope of $BP = \frac{1-1}{3-4} = 0$

Required equation of the line passing through points $B$ and $P$ is

$y - y_1 = m\left(x - x_1\right)$

$y - 1 = 0(x - 3)$

$y = 1$

70. $\because$ Line passes through point (2, -5) and intercepts of -3 on y-axis.

i.e. through point (0, -3)

Slope(m) = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-3-(-5)}{0-2} = \frac{-3+5}{-2}$

= -1

Equation of Straight line having slope -1 and y-intercept -3 is given by slope-intercept form.

i.e, y = mx + c

y = (-1)x + (-3)

y + x + 3 = 0

Hence, the required equation of line is y + x + 3 = 0.

71. Given, lines are

$2x + y - 7 = 0$ ... (i)

$x + y - 4 = 0$ ... (ii)

$2x - y - 8 = 0$ ... (iii)

From Eq. (i), $y = -2x + 7$

On comparing with $y = mx + c$, we get

$m = -2$

Since, parallel lines have equal slope.

$\therefore$ Slope of line parallel to $2x + y - 7 = 0$ is also -2.

Now, on solving Eqs. (ii) and (iii), we get

$x = 4$ and $y = 0$

$\therefore$ Intersection point of lines

$x + y - 4 = 0$ and $2x - y = 8$ is $\left(x_1, y_1\right) = (4, 0)$

Now, the equation of a line have slope $m$ and passing through the point $\left(x_1, y_1\right)$ is given by

$\Rightarrow y - y_1 = m\left(x - x_1\right)$

$\Rightarrow y - 0 = -2(x - 4)\left[\because x_1 = 4, y_1 = 0 \text{ and } m = -2\right]$

$\Rightarrow y = -2x + 8 \Rightarrow 2x + y = 8$

72. Slope of line 5x - 3y + 2 = 0 is

$= -\frac{\text{coefficent of } x}{\text{coefficent of } y}$

$= \frac{-5}{-3} = \frac{5}{3}$

Slope of line 6x - py + 7 = 0 = $\frac{-6}{-p} = \frac{6}{p}$

Lines are perpendicular each other

$\therefore$ Product of slopes = -1

$\frac{5}{3} \times \frac{6}{p}$

3p = -30 $\Rightarrow$ p = -1

$\therefore$ p = -10

Given line 6x - py + 7 = 0 is 6x + 10y + 7 = 0

Equation of the line, parallel to 6x + 10y + 7 = 0 is 6x + 10y + k = 0

line passes through (-2, -1)

$\therefore$ 6(-2) + 10(-1) + k =.0

-12 - 10 + k = 0 $\Rightarrow$ k = 22

Equation of the line parallel to 6x + 10y + 7 is 6x + 10y + 22 = 0 or 3x + 5y + 11 = 0.

73. The equation of a line passing through point p($x_1$, $y_1$) and slope 'm' is given by

y - $y_1$ = m(x - $x_1$)

Here P(9, $\frac{1}{2}$) and m = $\frac{-3}{2}$

Putting these values in above equation we get

$\Rightarrow y - \frac{1}{2} = \frac{-3}{2}(x - 9)$

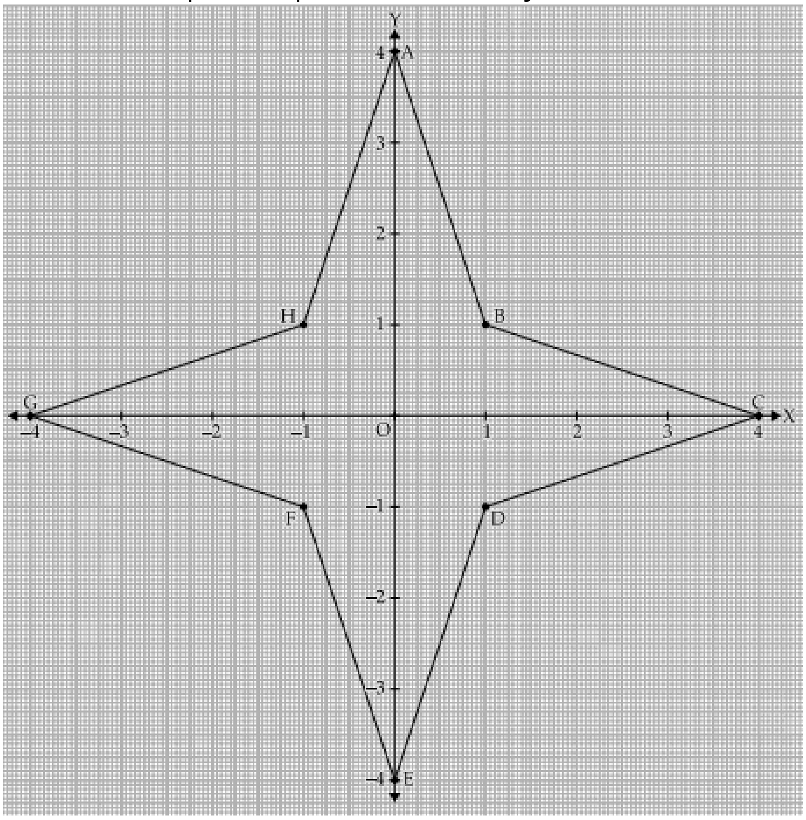
$\Rightarrow y - \frac{1}{2} = \frac{-3}{2}x + \frac{27}{2}$

$\Rightarrow$ 2y - 1 = -3x + 27

2y + 3x - 28 = 0

Hence, the required equation of line is 2y + 3x - 28 = 0.

74.




(c) $F(-1, -1)$

(e) Star

75. i. $M_x(3, 4) = (3, -4)$ $\therefore$ coordinates of $P'$ are $(3, -4)$ Clearly the eqn. of $PP'$ be $x = 3$

$\therefore$ The reflection of $O(0, 0)$ in the line $x = 3$ be $(6 - 0, 0)$ i.e. $(6, 0)$

$\therefore$ Coordinates of $O'$ are $(6, 0)$.

ii. $PP' =$ distance between $P(3, 4)$ and $P'(3, -4) = \sqrt{(3-3)^2 + (4+4)^2} = \sqrt{0 + 64} = 8$

$OO' =$ distance between $O(0, 0)$ and $O'(6, 0) = \sqrt{(6-0)^2 + (0-0)^2} = 6$

iii. $PO = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

$OP' = \sqrt{(3-0)^2 + (-4-0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

$O'P' = \sqrt{(3-6)^2 + (-4-0)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ units

$O'P = \sqrt{(3-6)^2 + (4-0)^2} = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = 5$ units

$\therefore$ perimeter of $POP'O' = PO + OP' + P'O' + O'P = (5 + 5 + 5 + 5)$ units $= 20$ units

iv. Here all sides are of equal lengths. $PP' = \sqrt{(3-3)^2 + (-4-4)^2}$

$= \sqrt{0 + 64} = 8$

and $OO' = 6 \therefore PP' \neq OO'$ $\therefore$ given figure forms rhombus.